X X

Brodsky High Energy Physics 1972 Vol. 5 -
Gatto Review on Tests of QED

Berezin 2nd quantization (1966)

S-Matrix references
Shirkov A.J.-Phys 31 755 (1963)
Atherley P.R. 125 745 (1962)
Stapp

Muon anomaly 1 part in 100,000 10 ppm.
Phys. Lett. 55B p. 420 (1975)

Barut: Electrodynamics and classical theory of fields and particles, 1964 ([illegible] classical)

Nobel lectures 1965 Schwinger, Tomonaga and Feynman [illegible] anecdotes

Hylleraas Q. Mechanics of 2-electron systems RMP 35, 421 (1963)

S. Klar. Nob. Sci 40 (1973) 194.
Statistical Correlation, Ergodic Theory.

Velo, Wightman: Constructive Q. Field Theory, 1973.

Lopes, Epistemological Studies in Contemporary Physics and Elem. Wartofsky (1974) Dordrecht.

Schwinger Ann. Phys 11, 1 (1960) (gauge theories)

G.E. Emch (1984)

Mathematical and Conceptual Foundations of 20thC Physics — North-Holland

Survey of Mechanics, Stat. Mechanics, SR, GR, QM and QFT from abstract mathematical point of view — not enough detail to learn from, but good to order for reference.

— refer to
Simms & Woodhouse Lectures on Geometric Quantization Springer Lecture Notes in Physics vol 53 (1976)

cf also M. ~~Foundatio~~ Bunge: Foundations of Physics (1967) — somewhat similar in style to Emch, but more philosophical

N. E. Hurt Geometric Quantization in Action — D. Reidel (1983)

Very clear and comprehensive discussion including the mathematical background

J.A. Eisele : Modern Quantum Particle Mechanics with Applications to Elementary Particle Physics (Wiley: New York) 1969.

gives very detailed manipulation of Dirac matrices etc, good discussion of parity violation in β-decay of Feynman's integral q.m. approach (to his disutility) and. particularly good on the Foldy-Wouthuysen transformation — all steps included and misprints corrected in the original papers refers to Kursunoglu. "Transformation of Relativistic wave equations" Phys Rev 101, 1419 (1956).

R. Shankar: Principles of Quantum Mechanics (Plenum: New York) 1980 gives particularly clear discussion of Feynman's path integral — defn of evaluation

G. Velo JA. Wightman (Springer) 1973 Constructive Quantum Field Theory report on Ettore Majorana Conference — detailed background to Shenley's 1976 course.

Combley (g-2) factors for muon, electron

Rep. Prog. Phys. 42 (1979) 1889.

very good up-to-date discussion of experiment, theory and relation to Washington experiments of Dehmelt et al.

Pauli RMP. 15 p. 175. (1943)
Vehling P.R. 48 55 (1935)
Pauli, Rose PR 49 (1936) p 462.

Kemble & Present P.R. 44 1031 (1932)*
Pasternack P.R. 54 81173 (1938) *
Williams P.R. 54 558 (1938)
Houston P.R. 51 446 (1937)
Retherford, Richardson & Williams P.R. Soc. 174, 164 (1940) *
Breit P.R. 71, 984 (1947)*

References from J. Cushing 'Some Aspects of Current Methodologies in the Maturity of Science as Reflected in Quantum Field Theory and S-Matrix Theory'

L.D. Landau 'On Analytic Properties of Vertex Parts in Quantum Field Theory'
Nucl. Phys: 13 (1959) 181-192

R.E. Cutkosky 'Singularities, Discontinuities of Feynman Amplitudes'.

Charap, Fubini "The Field Theoretic Definition of the Nuclear Potential — I".
N.C. 14 (1959) 540-559.

D. Atkinson 'A Proof of the Existence of Functions that Satisfy Exactly Both Crossing and Unitarity — I. Neutral Pion-Pion Scattering — No Subtractions'. ~~A.R.~~ Nucl. Phys. B7 (1968) 375-408.
(Schauder fixed point theorem in SM.T.;)

Stapp "Bell's Theorem and World Process"
N.C. 29 B (1975) 270-276.

P.W. Higgs Phys. Lett. 12 (1964) 132.
Phys. Rev. Lett. 13 (1964) 508.
P.R. 145 (1966) 1156.

Breakdown of Goldstone's Theorem for Gauge Theories

L. O'Raifeartaigh "Hidden Gauge Symmetry"
Rep. Prog. in Physics 42 (1979) 159–223.

Goldstone N.C. 19 (1961) 154

Goldstone, Salam, Weinberg P.R. 127 (1962) 965.
Proof of Goldstone's Theorem.

Cheng "Hadron Bootstrap Hypothesis"
P.R. 4D (1971) 2330-2335.

P. Collins An Introduction to Regge Theory and High Energy Physics — CUP 1977 — very good updated account of his earlier reports on the subject.

G. Velo and A. Wightman (eds): Constructive Quantum Field Theory: Lecture Notes in Physics vol. 25 (Berlin: Springer-Verlag) 1973.

mainly devoted to Euclidean field theory and $P(\Phi)$ FT models

good discussion of Probability theory of stochastic processes by Reed in [illegible] also [illegible] work of Nelson (1972 onwards). Euclidean [illegible] -tric field and by Dyson on [illegible] renormalization

J. Glimm and A. Jaffe Physics of Quantum Field models p. 133 – 198.

they gave the following basic references:

G.J.: $\lambda(\Phi^4)_2$ without cutoffs I. P.R. 176 (1968) 1945

II. Ann. Math. 91 (1970) 362

III Acta Math 125 (1970) 203

IV J. Math Phys. 13 (1972) 1558.

Also Comm. Math. Phys. 22 (1971), 1.

See Wg 'What is Renormalization?' in Seminar on partial diff. Eqs. Berkeley 1971. (Am. Math. Soc.)

See also - Quantum Field Models in Stat. Mech. and QFT ed. de Witt, Stora 1971

and Borel Quantum Field Models in Streater (ed) 'Problems of Contemporary Physics' 1972.

last paper on Euclidean Fermi fields by Osterwalder — and Fröhlich seems to be very useful.

J. Mehra ed. Physicist's Conception of Nature
1973 — dedicated to Dirac —
first articles on history of QED — earliest
on recent to Jordan for idea of
2nd quantization.
Lead article by Wightman on renormalizability
— most recent of Glimm, Jaffe
on infinities in exact soln of
model theories. (p 430)
Glimm, Jaffe 1969–1972.
solve (2+1) dimension theory with
ϕ^3, ϕ^4, $\Sigma\, P(\phi)\,\phi$ without renormalization
Infinities appear in the exact solutions
as well as in perturbation theory. So it
is not perturbation expansion which is at
fault. cf Glimm, Salam, Strathdee
P. R. D. 5 2548 (1972)
cp Glimm, Jaffe Proc. Enrico Fermi
School of T. ed Jost (1969) (course 45?)

Tachyons

Introduced by Bilaniuk, Deshpande, Sudarshan
in Am. J. Phys. 30 (1962) 718

~~Confusions discussed by~~ Causal anomaly discussed by

Pirani P.R. D1 (1970) 3221 this paper contradicted by Parmentola & Yee P.R. D4 (1971) 1912

Confusions treated by

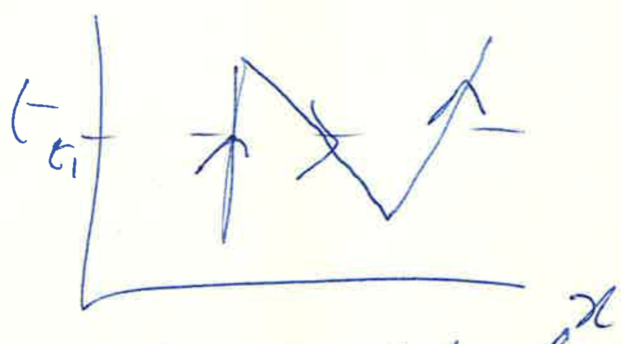
Feinberg P.R. 159 (1967) 1089.

they use 1 frame to describe complete signalling arrangement. See Root

Travelling Backwards in Time

R. Weingard. Synthese 24 (1972) p117

analyses Putnam's claim (J. Phil. 59 (1962) 658) that time travel is a conceptual possibility



are 3 electrons occurs at t_1, or one occurs

Idea is clearly adopted from Feynman. Is this possible up backward causation
W. says this is quite a different question.

Goldhaber & Smith a review article [illegible]

[illegible] [illegible] Rep. Prog. Phys. 1975

~~[illegible]~~ old. Can [illegible] of [illegible] [illegible]

& [illegible] by [illegible] [illegible] ~~[illegible]~~ [illegible]

[illegible] [illegible] [illegible] [illegible] [illegible], [illegible] [illegible] [illegible]

in time — [illegible] seems to be

[illegible] 1°- [illegible] [illegible]

[illegible] [illegible] position.

K. Schrader-Frechette

Atomism in Crises: An analysis of the current High energy paradigm. Phil. Sc. 44 (1977) pp 429-44[illegible]

argues for Kuhnian Crisis because

1.) old paradigm is unclear

2) paradigm fails to support normal problem solving research.

very muddled and confused paper on many points of detail.

J.C. Graves and J.F. Roper Phil. Sci. 32 (1965) 39.
'Measuring Measuring Rods'
argue as does ~~Ween~~ Putnam that Feynman interpretation is a natural one.
1) mysterious (unseen) processes are eliminated 2) reduced no. of entities 3) explains identity of e^+, e^-

But J. Earman 'On going backwards in time' Phil. Science 34 (1967) 211.
argues against Feynman for following reasons. ~~the~~ ~~the~~
1.) e^+, e^- are not totally similar since c invariance is broken by weak interactions
2.) annihilation, creation are not explained but annihilation is not into nothingness but into a photon

References to check:

Wheeler, Feynman: Direct particle interaction — RMP 21 (1949) Absorber Theory; 17 (1945) — Feynman RMP 20 (194[illegible] (Lagrangian theory)

Feynman P.R. 74 (1948) p. 939.

E.C.G. Stueckelberg - Helv. Phys. Acta. 14 (1941) p588, 15 (1942) p.[illegible]

J. Earman. Aust. J. Phil. 50 (1972) (Tachyons)

Synthese 24 (1972) Causal anomalies

Dummett. Phil. Rev. 1964 73 p. 338 Retrocausation

Chisholm, Taylor Analysis 20 73 (1960) "

Gödel RMP 1949 21 p 447 & Einstein Schilpp Vol.

Reichenbach Direction of Time p 266 (comment on Feynman).

Earman ✓ Phil. Sci. 34 (1967) p. 211 objects to Feynman's t→−t relabel det — Retrocausation

Weingard Space, Time, Geometry 1973 ed. Suppes closed time-like curves

Wainer ✓ Phil. Sci. 37 (1970) p. 81, 37 p. 223

Grünbaum et al. ✓ Phil. Sci 36 (1969) — [illegible] symmetry

Bonnor, Ellis ✓ Phil. Sci 34 (1967) pp 116.

Earman ✓ Phil. Sci 41 (1974) (Backward Causation)

Fitzgerald Boston Studies (Reference)
Vol VIII (1971)

Pirani P. R. D, 1, 3224 (1970) (")

Peres, Schulman Int. J. Theor. Phys. 6, 377 (1972) (")

Earman ✓ Covariance, Invariance, Frames & Relevance. Foundations of Physics 4 (1974) 267

Abarbanel ✓ R M P. 48 (1976) 435 Big Review! ✓

Graves, Roper ✓ Phil. Sci 32 (1965) 39 Feynman & others are noted

Fierz, Weisskopf → Pauli vol. Physics in 20th Century

Salam, Wigner → Dirac vol. Aspects of Q. Theory.

Schwinger Q.F.D. papers.

R.M.P.

Table of particles

April 1976

Bosons	Fermions
pions 150	Baryons: Nucleons {p, n} 950
kaons 500	Hyperons: Λ 1100
η 550	Σ 1200
} Mesons	Ξ 1300
	Ω. 1650
+ resonances	+ resonances
ρ 750	Δ (1232) 1250
ω 800	
J/ψ 3,100	

⟸ Hadrons ⟹

Bosons	Leptons
photon 0	neutrino, ν_e, ν_μ 0
graviton (?) 0	electron 1/2
W-particle (?)	muon 100
gluons (?)	quarks (?)

32 mesons
46 baryons
78

with antiparticles ~ 150 particles

Mass to 50 MeV.

Physics Reports

Regge Phenomenology. Irving, Worden. 34 (1977) 117–231

Colour Models of Hadrons Greenberg, Nelson.
32, (1977), 69–121
overview of paraquarks, coloured quarks

Quantum chromodynamics W. Marciano and H. Pagels.
36 (1978) 137–276.

Name QCD due to Gell-Mann
"confinement makes colour degrees of freedom an ~~it~~ in principle unobservable". It would be an irony of quantum mechanics, founded on an insistence on operationalism, produced a theory in which the fundamental entities were non-empirical mathematical constructs". Very clear discussion of modern theories — Excellent overview of modern state-of-play.

Gives references on conformal, scale invariance (+ discussion of renormalization proof)

Coleman, Jackiw Ann. Phys. 67 (1971) 552
Mack Nucl. Phys. B5 (1968) 499
Mack, Salam Ann. Phys. 53 (1969) 174.
Mostrup. P.R. 142 (1966) 1060.

Reasons for Q.C.D.

colour statistics

scaling properties

quark confinement mechanism — asymptotic freedom.

renormalizability

Phenomenological QCD — should come out of basic theory

(1) Parton model.

(2) Bag model for confinement

(3) potential models.

(4) string model — glue forms flux tubes between the quarks.

Detailed discussion of renormalizability, renormalization group etc

3 sorts of regularization

1) Pauli-Villars

2) lattice

3) dimensional

Exactly soluble 2-dimensional gauge theories

Detailed discussion of coloured states 2

classical solution — could be confined
in path-integral formulation of quantization

Also This Q. no could be violated as
a colour singlet divides into coloured
states — possible confinement mechanism

Conclusion QCD provides a potentially
rigorous QFT to justify the
application of quark, parton models.
— there is "dogmatic optimism that a
theory of strong interactions has been
found"!.

But QCD is very plausible.
the right questions have been put
in by hand. If they fail it
will be on dynamical grounds.

Also free quarks may exist at sufficient
high energies

Prospect of a unified theory of all fundamental interactions — weak, e.m. strong and gravitational.

Quantum geometrics
gravitons (Bose
gravitinos fermions)

Quantum chromodynamics
quarks — coloured gluons

Quantum flavour dynamics
leptons, quarks and weak flavour
flavoured bosons (W-particles)

Three flavour gauge field — 4 flavours - SU(4)
and 3 colours (SU(3)) + up, down,
red, white, blue. strange, charmed.

Vector gauge field of 8 coloured gluons -
(do not carry flavours).

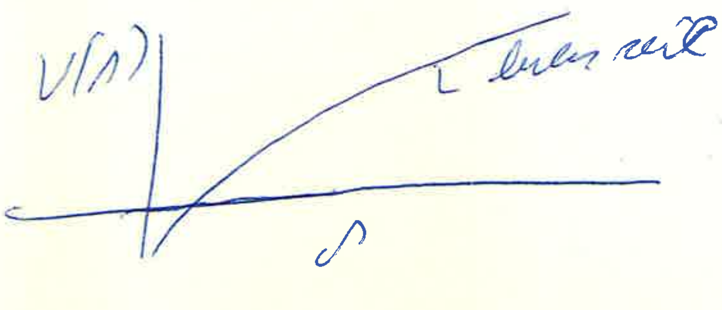
Colour symmetry is local (gauged)

Colour singlets exist which are bound states of gluons - known as glueballs or gluelike hadrons (they are flavour singlets)

quark confinement

In QED with abelian gluon, e-p force is attractive Coulomb law. field spreads out over all space.

In QCD with non-abelian gluons we might suppose self-interacting character of the gluon may imply a lower energy configuration is a tube of coloured flux between the quark and antiquark. The rise in the field energy in P.E. for such a configuration is a linear function of the separation distance (constant force), the quarks are confined, i.e. energy required to separate a colour singlet state into a coloured state is infinite



More recent renormalization procedure is due to Bogoliubov, Parasiuk, Hepp and Zimmermann (BPHP)

References in Physics Reports

C.S. Sharma: Correlation energy in atoms — 26C July 1976. Good critique of atomic calculations.

D. Rogovin, M. Scully Superconductivity & macroscopic quantum phenomena. 25C June 1976. Good account of macroscopic wave functions

R.M. Dicke? - Semiclassical & Electrodynamical effects in N.R. radiation theory 25C May 1976 — Good account of Jaynes - Neo classical theory - (cannot account for radiative corrections EPR correlation effects)

RMP Bohr, Mottelson, Rainwater on Collective motion in nuclei, + discussion of Elliott model 48 (1976) no 3.

Refer to Elliott 1958 P. R. Soc. A 245, 128 & 562.

Additional reference of [illegible]

Ohnuki & Kamefuchi "Para-Fermi Field Theory"
P.R. 170 (1968) p1279.

discuss relation of coloured quarks to paraquark models of hadrons.

Harle & Taylor P.R. 178 (1969) p2043.

cluster decomposition exp: separated systems may be treated as isolated systems.

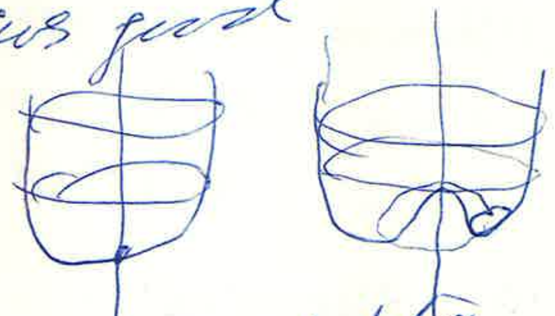
Springer Tracts in Modern Physics

Lennon - 2 & 3 nucleon systems
1976? - good account of alternative schemes.

Physics Today March 1976 Heisenberg "The Nature of Elementary Particles" p32

Physics Today
June 1975

S. Weinberg "Light as a fundamental Particle" — Good discussion of question — Is light a fundamental particle. — discusses Little group goes from SO(3) to E(2) as vel. of particle tends to c. — gives good illustration of broken symmetry



with indented bottom of bowl circular trough is stable. Position of ~~one~~ ball corresponds to vac. expectation values of a scalar field, oscillation frequencies to scalar particle mass and the nearly undamped rolling mode to a Goldstone boson of zero mass (can be excited with zero input of energy). i.e. frequency of mode is ∞).

Gauge group	Vector field	Theory
$U(1)$	A_μ	Q.M.
$U(2)$	$A_\mu, W_\mu^{\pm}$	Y.M.
$U(2) \otimes U(1)$	$A_\mu, \underbrace{W_\mu^{\pm}, Z_\mu}_{\text{pick up mass by Higgs-Kibble mechanism}}$	

Weinberg is enthusiastic for gauge symmetry as fundamental principle of nature that governs interactions.

Scientific American

Y. Nambu: The Confinement of Quarks.
Nov. 1976. p. 48.

(1) Discusses colour gauge symmetry with asymptotic freedom — refers to Politzer & Gross & Wilczek.

(2) String model.

3) Johnson MIT Bag model

4.) Wilson lattice-theory model.

K. Wilson P.R. D 10 2445 (1974)
Proposes his lattice gauge model & quark confinement.

Kogut & Susskind P.R. D9, 3501 (1974)
discusses physical picture of quark confinement in gauge theories (cf. Nambu article)

refers to Weinberg PRL. 31 494 (1973)
Carter ~~Corber~~, Rogers & Susskind P.R.L. 31 792 (1973)
P.R. D9 206 (1974)

for a general idea
of quark confinement via gauge theory

Weinberg (1973)
refers to Gross & Wilczek PRL 30 (1973) 1343
& Politzer PRL 30 (1973) 1346

for crucial ideas on quark confinement &
asymptotic freedom in colour gauge theories

In 1973 Gross, Wilczek, and Politzer
show how ~~colour gauge theories symmetry~~ may
~~account for~~ scaling, confinement properties of
quarks - may result from colour gauge
symmetry ('chromodynamics)

Glashow, Iliopoulos & Maiani (1970)
introduced charm

De Rújula, Giorgi and Glashow (1975)
develop colour gauge symmetry as theory of
quark interactions (chromodynamics)

1973 Gross and Wilczek, and Politzer
explain scaling, confinement properties of quarks

in terms of [illegible] [illegible] quality (chemqualities)

Gribov et al. (+ Putnam) 1974
Y. Nambu: "The confinement of quarks" Sci. American
Nov 76. p. 48.

describes interaction between quarks (coloured) by non-abelian gauge field of massless vector gluons (of colour-anticolour pairs)

Confinement due to (1) asymptotic freedom at ultra-violet freedom as opposed to infra-red slavery (Politzer & Gross, Wilczek). force ~~increases~~ remains constant level defined as effective colour charge increases with distance

(2) string model: quarks attached to ends of massless rotating strings
cannot exist, preq. theory near Nielsen & Olesen 'string' solutions
(3) MIT Bag due to Kenneth Johnson.
(4) K. Wilson: lattice model of space-time quarks on lattice sites, colour gauge field propagates along st. lines (strings) joining them.

Reference for bag model Johnson et al.
P.R.D. 9 (1974) 3471, P.R. D 12 (1975) 2060

Colour gauge theory of gluons due to de Rujula, Georgi, Glashow
P.R.D 12 (1975) 147.
conforms Weisskopf's quark philosophy
"Many recent theoretical, experimental developments seem to confirm the latter philosophy"

Note colour gauge theory is not spontaneously broken, gluons are massless, but are source of colour field – so gluons are also confined

"Once upon a time there was a controversy in particle physics. There were some physicists (Eclectic) who denied the existence of structures more elementary than hadrons, and searched for a self-consistent interpretation wherein all hadron states, stable or resonant,

Agnese & Calvini P.R. 12 (1975) 3800

"Gauge fields arising from space-time symmetries and gravitational theories"

refers to Utiyama & also

Kibble J. Math. Phys 2, 212 (1961)

for extension of Utiyama to local Poincaré covariance.

⇒ were equally elementary; others (e.g. Gell-Mann) appalled by the teeming democracy of hadrons insisted on the existence of a small number of fundamental constituents and a simple underlying force law. In terms of these new fundamental things, hadron spectroscopy should be qualitatively described and essentially understood just as are atomic and nuclear physics"

R. Roman: Introduction to Quantum Field Theory. John Wiley & Sons. Inc. New York (1969)

Part I discusses Lagrangian Field Theory

concludes with discussion of renormalization theory in terms of Schwinger's functional methods (series)

Part II deals with LSZ, Wightman axioms, Haag's theorem, analytic and finishes with axiom' algebra

p. 466 " Therefore it may be possible, in principle, to determine the renormalization e_s, e_u, e_t by some self-consistent iteration method applied to the system of the three (coupled) dispersion relations. The feasibility of such methods depends on whether the [illegible] Mandelstam amplitude $D(s,t,u)$ considered as a function of the three complex variables is an analytic function in the $s,t+u$ variables [illegible]. This analytic requirement, in turn, [illegible] need stronger than analytic in each variable separately"

Quantum Gravity : An Oxford Symposium.
ed. C.J. Isham, R. Penrose, D.W. Sciama

Good general introduction by Isham.
refers to Brill & Gowdy Rep. Prog. Phys. 33 413 (1970)

also Wheeler ed: Magic without magic:
Freeman (1972)

More recent article is:—
Ashtekar and ~~Geor~~ Geroch
Quantum theory of gravitation
Rep. Prog. Phys. 37 (1974) 1211-125[illegible]

For superspace (space of all equivalence classes of metrics (connected by smooth transformations)
i.e. a point in superspace is a whole geometry – quantum fluctuations act in superspace not a fixed point in superspace as in ~~the~~ classical physics
cf also Carmeli, Fickler, Witten: eds Relativity 1970.

References for post 1953 renormalization theory

Matthews and Salam P.R. 94, 185 (1954)
(elimination of zero fields)

Redmond
(essential singularities in propagators)
P.R. 112 1404 (1958)

Redmond & Uretsky Annals of Physics 9 106 (1960)

Dyson Proof of asymptotic expansion
P.R. 85 631 (1952)

cf also Hori Prog. ~~Theor.~~ Phys 8 569 (1952)
Riddell. P.R. 91, 1243 (1953)

References for [illegible].

Kallen. Helv. Phys. Acta 25, 416 (1952)
Lehman N.C. 11 342 (1954)
Stueckelberg & Petermann Helv. Phys. Acta. 26 499 (1953)

Gell-Mann & Low P.R. 95, 1300 (1954)

cf. Streater references:—
Jaffe Comm. Math. Phys. 1, 127-49
2, 301-26
Hepp J. Math Phys. 6 1762 (rotation invariance paper)
Hurst Proc Camb. Phil. Soc. 48 (1952) 625
Hurst. Proc Roy Soc A 214 (1952) 44 divergence of perturbation series, Thirring Helv. Phys. Acta 26 (1953) 33-52

References from Bartas

LSZ. NC 1, 205 (1955), 6, 319 (1957) (in English)

Haag. Mat Fys. Medd 29 no 12 (1955)

Hall & Wightman P.R. 101, 860 (1956)

Wightman Mat Fys Medd. 31 no 5 (1957)

Greenberg P.R. 115, 706 (1959)

Schweber, Wightman & Schroder P.R. 98, 812 (1955)

Dispersion Relations

Hamilton Prog in Nucl. Phys. 7. (1959) 16
(Early history)

Axiomatic S-Matrix Theory:
Symanzik J. Math Phys. 6, 827, 845, 852 (1965) 22
Olive P.R. 135 B 745 (1964) 23
Stapp P.D. D 3 (1971) (~~1971~~) 1303. 24
Rondelstam. ~~N.C. 15 658 (1960)~~
Ann. Phys 21 302 (1963) 18
(CDD poles and analytic 2nd sheet.)
Reps Prog. Phys 5 (1962) Review article p. 99-162 17
Iagolnitzer and Stapp (macrocausality → analyticity) Comm. Math Phys. 14 15 (1969)

Gale: Chen's Mereology: J. Hist. Ideas
35 (1974) 339.

Wesley & Rich P.R.L. 24 (1320) 1970.
Lerner & Wright P.R. D 8 3171

Gell-Mann, Goldberger & Thirring P.R. 95 1612 (1954)

Toll - P.R. 104 1760 (1956)

Goldberger P.R. 97 508 (1956)
99 979 (1956)

References from Jost

S-Matrix Philosophy Brenig & Haag
Fortschr. Physik 7 (1959) 183

Verification of dispersion relations:-
Noyes & Edwards P.R. 118 (1960) 1409.

Mandelstam on analyticity from perturbation theory
P.R. 112 (1958) 1344
115 (1959) 1741

from axiomatic field theory
N.C. 15 (1960) 658.

Some papers of Heisenberg on S-Matrix
Z. Physik 120 (1943) 513, 673. 1944

CGLN on π-N system.
P.R. 106 (1957) 1337

Chew & Low P.R. 101 (1956) 1570, 113 (1959) 1640.

Low P.R. 97 1392 (1955)

[illegible] P.R. 102 (1956) 1174, 100 (1955) 1503

Alles & Zemach P.R. 131 2305 (1963) NN* Isobar
Zachariasen & Zemach P.R. 128 849 (1962) ρ bootstrap.

Regge (1959) N.C. 14 951

Chew & Frautschi P.R.L. 7 394 (1961)

Chew & Mandelstam P.R. 119 467 (1960)
Chew, Frautschi & Mandelstam P.R. 126 1202 (1962)
Hamilton & Woolcock R.M.P. 35 (1963) 737
(Cutkosky calcn)

not relevant → Donnachie Rep Prog. Phys. 36 (1973) 695 πN review
Eden " " 34 (1971) 995 Regge review

Donnachie Hamilton & Lea P.R. 135 B515 (1964)
Donnachie & Hamilton Ann. Phys. 31 410 (1965)
D. Bailin (weak interaction theory) Rep Prog. Phys (1972) 34, 491

Kronig: J. Optical Soc. America (1926) 12 547 (at borders)
~~Kramers:~~ Kronig: Physica 12 543 (1946)
Korff & Breit R.M.P. 4 471 (1932) (no derivation of dispersion relations)
refer to Wolf & Herzfeld Handbuch der Physics
Kramers. Atti Congr. Intern. Fisici Como 2 545 (1927)

Zohar B.J.P.S. 24 (1973) p. 95.

The double heuristic role of mathematics in Science

2 ways physics has heuristic function in stimulating mathematicians (cf invention of calculus or vector analysis)

What about reverse process.

(1) maths may modify a physical principle
cp M's route to Maxwell's equations -
& his introduction of the displacement current.

(2) We may seek to find a realistic interpretation to mathematical quantities used in peculiar theory.

eg. Lorentz → L.F.C. as a real dilation contraction not just a mathematical transformation
→ M.F.H from which L.F.C. follows

or Dirac → -ve energy soln
→ positrons.

Stapp H. P. P. R. 125 (1962) 2139 – 2162

"Derivation of the CPT Theorem and the Connection between Spin & Statistics from Postulates of the S-Matrix theory"

Comments on analyticity postulate:

"Maximal analyticity means that only singularities in the physical sheet are those required by unitarity (these include the usual anomalous and other singularities). This analyticity requirement evidently contains some extension of the usual locality requirement. ... The postulated domain of analyticity may [however] be larger than the of that can be deduced from field theory.

In this case the two theories would be different mutually incompatible systems. Indeed there is the expectation: the apparent inconsistencies of field theory are expected to be removed, as a result of weakening the locality requirement."

Chew G. F. & Frautschi S. C. P.R.L. 8 (1962) 41–44.

" Regge Trajectories and the Principle of Maximum Strength for Strong Interactions":

Explain the Principle provides part of view of Regge theory.

" all of strong-interaction physics would flow from:

(a) the Principle of maximal analyticity of the S-matrix in angular as well as linear momentum

(b) the Principle of Maximum Strength

(c) the Conservation of B, S & I. There should be no arbitrary dimensionless parameters and apart from the constant of light (or mass) to be added to ħ & c; there are no elementary particles"

C & F believe (b) & (c) may follow from (a).

References for [illegible]

Old view of renormalization:

Källén Helv. Phys. Acta. 25 416 (1952)

Lehmann N.C. 11, 342 (1954)

Gen. view:

Stueckelberg, Petermann Helv. Phys. Acta 26, 499 (1953)

Gell-Mann, Low. P.R. 95, 1300 (1954)

Critique of field theory:

Landau et al. N.C. 13, Suppl 3, 80 (1956)

& Niels Bohr, development of Physics, 1955

Dispersion relations

Mandelstam Rept. Prog. Phys 25, 99 (1962)

Critique of [illegible] in

V.B. Adamskii: Soviet Phys.-Usp. (English translation) 4, 607 (1962)

Streater & Wightman PCT, Spin & Statistics, all that (1964)

~~Chapter I~~ In Intro. S & W. refer to Dirac, Jordan, Heisenberg, Pauli formulation of rel. QFT. Consistent description not expected, since theory arose by quantizing a classical theory containing as e.m. united of point particles.

Main Problem of QFT is to kill it or cure it.

Reference to the Field version (Field Society) cf. Shakers, New England sect who built "a non-scientific, solid barns & led celibate lives, ~~of~~ of proving rigorous theorems & calculating no cross-sections".

Chapter I non-relativistic ideas simple & not actual ones or not physically realizable. Hilbert space of states contains coherent subspaces.

Symmetry operators defined à la Wigner (rigorous theorem not proved)

— Poincaré of Lorentz, Poincaré Groups

Chapter 17

Mathematical [illegible]

A type of distribution or generalized class of function $T(f) = \int T(x) f(x)\,dx \quad \forall f \in S$

S is set of test functions.

$T(f)$ is a distribution (continuous linear functional on S)

$T(x)$ is a function

Tempered distributions.

f is complex-valued, ∞ differentiable which, and derivatives, $\to 0$ at ∞ faster than any power of the Euclidean distance.

Derivative of a distribution $\frac{\partial T}{\partial x}(f) \stackrel{\text{Df}}{=} -T\left(\frac{\partial f}{\partial x}\right)$

[illegible] [illegible]

Laplace transform $g(u) = \int_0^\infty e^{iuh} f(h)\,dh$

is a boundary value of a holomorphic function viz the Laplace transform

$$g(x+iy) = \int_0^\infty e^{i(h(x+iy))} f(h)\,dh.$$

which is holomorphic in upper half-plane $y > 0$

W. generalizes this to functions of several complex variables. 2

Thm. 2.4. $F(C^n)$ is holomorphic iff it is (p51)
continuous in all variables together, holomorphic in each variable separately

— not necessary by Hartogs' theorem
cf. Bochner, Martin ch. VII

Hartogs' theorem not true for diff functions.
(cf Mandelstam ?)

cf p 82 discussion of non-separable Hilbert spaces.
i.e. ∞ tensor product of separable H-spaces.
(∞ no. of degrees of freedom ⇏ non-separable)
— also [illegible] [illegible] in 1st [illegible]

Ref: A.S. Wightman: Analytic functions of several complex variables pp 159–221,
Dispersion Relations, Elementary Particles,
Wiley, New York (1960)

Ch. 3 deal with fields, vac. expectation values. — observables are smeared operators (cf Bohr, Rosenfeld) i.e. $E(x,t) \to E(f)$

function distribution

assumes microscopic causality or local commutativity.

What about covariant quantization:
not observed in terms of operators smeared over space and time.
we assume instead smeared fields give all observables in some.
any state is derived from vacuum by operating with polynomials in smeared fields $P(\phi_1(f), \phi_2(g) \dots)$

We do not discuss scattering — relies on asymptotic conditions to Haag, Ruelle assure $H = H^{in} = H^{out}$ (cf Jost's book)

vacuum expectation values we expand

$(\Psi_0, \phi_1(x_1) \ldots \phi_n(x_n) \Psi_0)$ [illegible] reconstruction

theorem we can always construct a field theory from a theory of Wightman functions. (i.e. [illegible] of vac. exp. values)

Ch IV deals with general theorems of

Rel. Q.F.T.

(1) PCT theorem.

(2) Spin-statistics theorem — theory of Klein transformations

(3) Haag's Theorem:

For interaction we assume, since $\phi(x,t)$, $\phi_{in}(x,t)$ satisfy same equal-time commutation relations,

$$V(t)\,\phi(x,t)\,V(t)^{-1} = \phi_{in}(x,t). \quad (1)$$

Time dependence of V reflects presence of interactions

$$S = \lim_{t\to\infty} V(t)\,V(-t)^{*}.$$

But there may be inequivalent ops of the commutation relations, so existence of L is not necessary, but possible. Haag's theorem says it is impossible unless $\phi(x,t)$ is itself a free field there is no V interchanging (?

In effect "Renormalisation gets mixed up with the dynamics in the sense that the dynamics determines which representation of the canonical commutation relations we use."

Holds equal time commutation relations make no sense, fields must be smeared in time as well as space.

Final section on Borchers classes

different fields which use same S-matrix etc

N.B. Ruelle "We have eliminated all theories where field is non-existent".

W. gives v. gd Bibliography:

PCT theorem 1st proved by Lüders (suggested by Zumino)
Dansk Mat. Fys. Medd 28, 5 (1954)

proved if P is a symmetry then CT is a symmetry

Pauli first proved PCT is always a symmetry
in Niels Bohr and the Development of Physics
W. Pauli (ed) Pergamon, 1955.

Spin-statistics due to Fierz (1939), Pauli (1940)

general proof due to Lüders, Zumino (1958)

Haag's theorem "On Q. Field Theory" Dan Mat Fys
Medd 29, 12 (1955)

generalized in
Hall, Wightman, M. F. Medd Dan. Vid Selsk.
31, 5 (1957)

Haag-Ruelle theory in
R. Jost, General Theory of Quantized Fields
1963

Note: on p16 WS refer to old-fashioned views of symmetry $A \to A'$ which preserved exp. values etc.

$\Rightarrow$ unitary transformations & rules

But anti-unitary symmetries also important – but dealt with by "direct physical way" of Wigner. But these can be observed under substitution rules which are "rather curious". (p17)

Word these as observable symmetries which are not physical symmetries.

Physical symmetry $\longrightarrow$ observable symmetry.

Does observable symmetry $\to$ physical symmetry

(probably yes?)

The anomalous magnetic moment of the muon
by Farley. Contemporary Physics (1975) 16 p 413
gives complete references
refer to Rich & Wesley R.M.P (1972) for
the electron anomaly 44 250
& Bailey et al. Phys. Lett. B, 55, 420 (1975) for latest
muon results.

References on Conformal Group

Salam & Mack Ann Phys (N.Y.) 53, 174 (1969)

Salam & Pacello Ann. Phys. 67 552 (1971)

S. D. Zoomin in Corncloft J. Math Phys. 5, 490 (1964)

look at Barut Electrodynamics & classical theory of fields & particles 1963

and Barut The Theory of the scattering matrix 1967
good solid account with detailed derivations
& good mathematical appendices

Callan PR D 2 1541 (1970)

Gross, Wilczek PR ~~D 8 3497 (1973)~~, D8, 3633

D9 980 (1974)

Sequel, less interesting

C

intro. to ~~[illegible]~~ Gross, Wilczek Fr HASF C. theory and my reference

Veneziano Phys Rep 9C no 4 (1974)

Good discussion introduction of dual models

Politzer Asymptotic Freedom: an approach to Strong Interactions: Phys. Reps. 14C no 4 (1974)

discusses gauge theories, renormalization group methods.

Mandelstam, Yan PRD 1. 1035, 1617, 2402

see field theory rules for partons. → 187, 2159 (1969)

parton-like scaling behaviour from local field theory.

Bjorken, Paschos: D 1 3151 (1970)

179 1547 (1969)

185, 1975 (1969)

Salam & Wigner Aspects of Quantum Theory ed.
(1972) Festschrift for Dirac. Bibliography of Dirac.

good article by Jost. "Foundations of quantum theory" refers to Wentzel.

note Dirac's 1932 paper offering Heisenberg & Pauli quantization of the electron field.

Proc. Roy Soc. A136 453 (1932)

– lead on to the Dirac - Fock - Podolski papers for quantizing a system of electrons

Also Rosenfeld Theory of the electron 1897–47)

> "We cannot [therefore] suppose the field to be a dynamical system on the same footing as the particles"

Heisenberg & Pauli   Z. f. Phys. 56 (1929) p. 1.

Introduction   In Q. theory it has not until now been possible to consider from a single viewpoint and to unite without contradiction all electrical & mechanical phenomena, above all magnetic interaction with the field, radiation interaction and so on. In particular it has not been possible to allow for the finite propagation speed of electromagnetic interaction correctly. The present work aims to fill these gaps.

For this therefore it will be essential to use a rel.-covariant formalism which allows us to treat interaction between matter & field as well as particle interaction. This problem appears [illegible] very difficult, on account of the complete [illegible] rel. generalization of the above problem, and we will reach a satisfactory solution and after an elucidation of these fundamental difficulties. Renormalization: the [illegible] effort can [illegible] be supported [illegible] — perhaps it can be solved by very careful [illegible] formulation. It is shown in [illegible] part whether the rel. formulation of many particle problem of Hamiltonian methods is not possible.

FJM Farley : the Status of QED

Rivista del Nuovo Cimento. 1 (1969) 59–86.

Corinaldesi Introduction to Dispersion Relations

N.C. Suppl. [10] 14 (1959) 3

(vol. includes article by Haag et al on axiomatic field approach

Pauli (ed). Niels Bohr and the Development of Physics.

includes Pauli on PCT

Luders "On the Quantum Theory of Fields"

Rosenfeld "On quantum Electrodynamics"

Dirac. PRS 112 (1926) 661. "On the theory of QM"

Discusses many-particle systems from F.D. statistics, B.E. equivalent to symmetry props of wave functions

p 666 "It would appear to be possible to build up an electromagnetic theory in which the potentials of the field at a specified point $x_0 y_0 z_0 t_0$ in space-time are represented by matrices of constant elements that are functions of $x_0 y_0 z_0 t_0$"

Dirac PRS 113 (1927) 621
"The physical interpretation of the quantum dynamics"

discusses general transformation ~~theory~~ and introduces the δ-function (refers to Lanczos' work on continuous matrices) — stresses commutativity of q-numbers & c-numbers.

Dirac. P.R.S. A 117 (1928) 610–624.

introduces the Dirac eqn — gives an explanation of "duplexity" phenomena. pure requirements of relativity & to general transformation theory".
Difficulties of Klein-Gordon eqn + Linearised wave eqn.
applies to hydrogen atom — brilliantly clear & simple paper.

Dirac P.R.S. A 118 (1928) 351–361..
(communicated 1 month later exactly)
concerning conserved current, selection rules, intensity formulae for multiplets & Zeeman effect.

Dirac P.R.S. A 126 (1930) 360–365.
"A theory of electrons & protons":

first says "electron with negative energy moves in an external field as though it carries a positive charge.
-ve negative energy electron related to a proton
— refers to Weyl for this suggestion Z. f. Phys. 56 (1929) 332.

we cannot simply assert −ve energy electron is a proton, otherwise electrons could turn into protons by direct transition conflicting with conservation of charge. & other difficulties

Dirac then proposes his hole theory.

Assume all negative energy states are filled except for a few holes.

"a hole in a region that is otherwise saturated with electrons is much the same thing as a single electron in a region that is otherwise devoid of them."

[illegible] ∞ electric field due to negative energy electrons — Dirac takes ρ in M's eqns as "departure from the normal state of electrification of the world". Dirac gives persuading argument for asymmetry in masses of electron & proton (this was later dispelled by Weyl). Ends paper by discussing the detailed description of intermediate states of −ve energy (intermediate states of Klein-Nishina formula is not something these states for Compton scattering) Here refers to possibility of electron-proton annihilation

Oppenheimer P.R. 35 (1930) p. 562–563. (letter)
raises a number of objections to Dirac's
hole theory of the proton.
He argues that scattering of light by electron &
proton in Thomson limit (long λ) should
give same results — actually in ratio of
square of masses.
Also calculates mean life of matter
against proton-electron annihilation
→ 10^{-10} secs. O. proposes protons
& electrons are 2 independent particles
& assumes negative energy states for
both particles are normally completely
filled (Note first reference to the
anti proton here)
detailed calculations on e-p annihilation
rate given by O. in P.R. 35 (1930) p. 939–947.
"Two notes on the probability of
radiative transitions".

Dirac PRS A133 (1931) p. 60—72.
"Quantized singularities in the electromagnetic Field":

" The steady progress of physics requires for its theoretical ~~foundation~~ formulation a mathematics that gets continually more advanced ... — modern physical developments have required a mathematics that continually shifts its foundations & gets more abstract. Non-Euclidean geometry & non-commutative algebra, which were at one time considered to be purely fictions of the mind & pastimes for logical thinkers, have now been found to be very necessary, for the description of general facts of the physical world ... advance in physics is to be associated with a continual modification & generalization of the axioms at the base of the mathematics rather than with a logical development of any one mathematical scheme on a fixed foundation."
[as ~~attend~~ was expected in last century].

"The most powerful method of advance that can be suggested at present is to employ all the resources of pure mathematics in attempts to perfect & generalize the mathematical formalism that forms the existing basis of theoretical physics, and after each success in this direction, to try to interpret the new mathematical features in terms of physical entities (by a process like Eddington's Principle of Identification)"

Dirac's sees his hole theory as exemplifying this methodology — a physical interpretation of the -ve energy states already necessitated by the formalism.

Refer to Weyl. Gruppentheorie und Quantenmechanik p234 (1931) pf proofs +ve particles have same masses. D. credits Oppenheimer with suggestion that all negative energy states are filled. "An encounter between 2 hard γ-rays (-- could lead to the creation of simultaneously (an electron) anti-electron"!

D- goes go to consider possible connection between smallest charge & smallest magnetic pole (Monopole)
Here's theory above for the existence of monopoles.
He says rather plaintively "Under these circumstances one would be surprised if Nature had made no use of it"
D concludes with question "why isolated magnetic poles are not observed. Particle forces of attraction between poles of opposite sign is so large that ~~thy~~ that" this very large force may perhaps account for why poles of opposite sign have never yet been separated"

Dirac P.R.S.A 136 (1932) p. 453–464.

Relativistic Quantum Mechanics

Dirac criticizes Heisenberg-Pauli Q.E.D. for not distinguishing particle, field "the field should appear in the theory as something more elementary & fundamental".

Referring to Heisenberg's claim to base QM on observables says "Strictly speaking, it is not the observable quantities themselves that form the building stones of Heisenberg's algebraic scheme, but rather certain more elementary quantities, the matrix elements, having the observable quantities as the squares of their moduli. The critical phase factors introduced in this way are essential (my italics))."

Dirac treats particles by 1st quantization then quantizes the e.m. field.

D. also criticizes H-P theory for "involving many quantities which are unconnected with results of observation and which must be removed from consideration if one is to obtain

a clear insight into the ~~unfortunately~~ underlying physical relations !!

<u>Fermi</u> R.M.P. 4 (1932) 87 - 132.
Treats Dirac's radiation theory
for <u>electr</u> revives Q.E.D. without
relativistic conditions - but
most discussion of photons - some
bibliography refers to Waller
Z. f. Phys. <u>62</u>, 673 (1930) for
calculation of self-energy of electron.

References
Lamb shift Expt

Lamb, Retherford P.R. 72 (1947) 241
75 (1949) 1325, 1332
79 (1950) 549
81 (1951) 222.
85 (1952) 259. 86, 1014 (1952)

Triebwasser, Dayhoff, Lamb. 89 (1953), 98, 106

Lamb. Rep. Prog. in Physics 14 (1951) 19.
Robinson, Shyn: P.R. 168 (1968), 4, P.R.L. 24 (1970) 559

Theory Kroll, Lamb P.R. 75 388 (1949)
French, Weisskopf 75 1240 (1949)
74, 1430 (1949)
correction 76 769 (1949)
Feynman

Fukuda, Miyamoto, Tomonaga Prog. Theor. Phys 4 (1948) 47, 121.

$\alpha(Z\alpha)^5$
Baranger, Bethe, Feynman. 92 482 (1953)
Karplus, Klein, Schwinger 86, 288 (1951)
Salpeter (recoil) ↑↓ 89, 92 (1953)

4th order α^2 Baranger, Dyson, Salpeter P.R. 88 680 (1953)
Bersohn, Weneser, Kroll, P.R. 86, 596 (1952)
91, 1257 (1953)

$\alpha (Z\alpha)^6$

Layzer P.R.L. 4, 580 (1960)
Fried, Yennie P.R.L. 4 583 (1960)

More accurate $\alpha^{?}$ initiated by ~~Brodsky~~ Applequist & Brodsky PRL 24, 562 (1970)

Electron anomaly

Theory
Schwinger 73 416 (1948)
Karplus, Kroll 77, 536 (1950)
Sommerfield P.R. 107, 328 (1957) — Ann. of Phys. 5, 26 (1958)
Petermann Nucl. Phys. 5 677 (1958) Helv. Phys. Acta 30, 407 (1957)
Kroll (unpublished) (1957.)

Expt. Hyperfine studies Nafe, Nelson, Rabi P.R. 71 (1947) 914.
Interpretation as anomalous moment
Breit 72 (1947) 984, 73 (1948) 1410 ✓ correction to power paper

Zeeman splitting Foley, Kusch P.R. 72 (1947) 1256
73 (1948) 412
74 (1948) 250.

1. Welton, P.R. 74 (1948) 1157 – 1167

Some observable Effects of the Quantum-Mechanical Fluctuations in the electromagnetic Field.

gives classical picture of zero point fluctuations in E, B giving rise to a $\langle(\Delta r)^2\rangle$ for free electron which gives a level correction

expected from $\langle V(r+\Delta r)\rangle_{av}$.

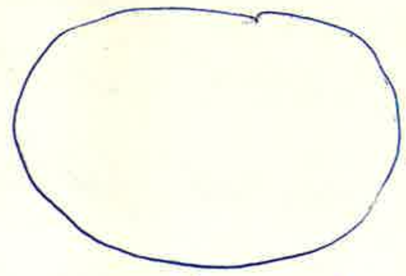
But effective magnetic moment due to angular motion of spin is of wrong sign – discrepancy due to magnetic interaction of electron with the field – zero energy states of the vacuum. (similar report to Lorentz.)

2. Kolsky P. Phys. Soc. 4 (1949) 319 – 330.

Semi-classical Treatment of the Radiative Corrections – I.

Explains correct sign of magnetic moment by allowing for Zitterbewegung of electron oscillations of electron velocity magnetic moment oscillation of position (Zitterbewegung) increases magnetic moment

Welton
effect.



increased
effective diameter
due to Zitterbewegung.
opposite sign
) twice magnitude
of Welton effect

Oppenheimer, J.R. "Note on the theory of the interaction of field and Matter". P.R. 35 (1930) p. 461 – 477.
discusses self energy of electron, concludes that levels and differences between levels are shifted by infinite amounts.

Pauli, W. "On Dirac's New Method of Field Quantization" R.M.P. 15 (1943) p. 175–[illegible]
discusses the Dirac-Wentzel λ-limiting process. In discussing self-energy Pauli says "It is the writer's opinion that this difficulty could be overcome only by using, instead of the λ-limiting process, a new, probably purely quantum theoretical method". Note that Pauli is referring to the electron self-energy in hole theory which is still logarithmically divergent even after Dirac's λ-limit and quantization with indefinite metric

<u>E.A. Uehling</u> "Polarization Effects in the Positron Theory" P.R. <u>48</u> (1935) p. 55 – 63.

Uehling discusses modification of Maxwell's equations due to vacuum polarization using the Heisenberg relativistic formalism (Heisenberg Zeits f Physik <u>90</u>, 209 (1934) following a suggestion of Dirac, Proc. Camb. Phil. Soc. <u>30</u>, 150 (1934).
Method is also equivalent to Furry & Oppenheimer P.R. <u>45</u>, 245 (1934)

Uehling discusses change in Coulomb law between two charged particles (<u>increased</u> repulsion. He discusses displacement of energy levels – both to <u>increased</u> separation of doublet lines in Balmer series which is opposite in sign & too small "by a factor of at least ~~Rn~~10".
refers to Kemble, Present (1933) re origins of the hydrogen anomaly.

Pauli & Rose M.E. "Remarks on the Polarization Effects in the Positron theory". P.R. 49 (1936) p. 462–

obtains simpler derivation of Uehling's result.

Interaction energy between charged particles

is $V(r) = z'z''e^2\left(1/r - \alpha/\pi\, U(r)\right)$

$U(r)$ falls off exponentially for $r \gtrsim \hbar/mc$

and is negative in sign, so $V(r)$

is greater than Coulomb.

$U(r)$ is function first calculated by

$U(r)$ also given by

$$\delta J(x') = \alpha/4\pi^2 \int U(r)\, \nabla^2 J(x'')\, dx''$$

$$r = |x' - x''|$$

we have omitted ∞ of term

$$\delta J(x) = \text{Const} \times J(x).$$

Kemble E.C. & Present R.D.

"On the Breakdown of the Coulomb Law for the Hydrogen atom:" P.R. 44 (1933) p. 1031–32.

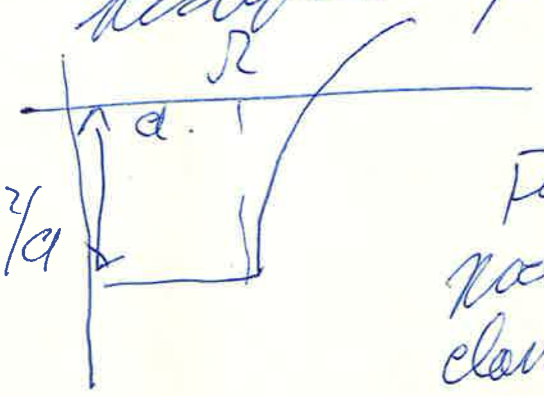
refers to anomalies in doublet separation in Balmer series for hydrogen from [illegible] measurements by Kent, Taylor & Pearson P.R. 30 266 (1927)

W.V. Houston, Astrophys. J. 64 81 (1926)

Spedding, Shane & Grace P.R. 44 58 (1933)

Houston & Hsieh Bull. Am. Phys. Soc. 8 no. 6, p. [illegible] (1933)

Consider modified potential



Find $a \approx 5 \times 10^{-12}$ cm much larger than classical electron radius 2×10^{-13}.

S. Pasternack "Note on the fine structure of H_2 and D_2". P.R. 54 (1938) p. 113.

refers to later work confirming discrepancy.

Houston & Hsieh P.R. 45 263 (1934)

Williams, Gibbs P.R. 45 475 (1934)
Kopfermann, Naturwiss. 22 218 (1934),
Spedding, Shane, Grace. P.R. 47, 38 (1935)
Houston P.R. 51 446 (1937)
Then work was by *Williams, R.C. P.R. 54 558 (1938)
confirm discrepancy in case of deuterium -
Pasternack argues energy for H_2 in
terms of a 2S level displaced
upwards by .030 cm^{-1}. "[This] would
seem to point towards some perturbing
interaction between the electron & the
nucleus". Too large to be due to
finite size of electron & proton.

* Williams refers to Bethe's suggestion
discrepancy was due to hyperfine structure
— much too small to account for
the discrepancy.

J.W. Drinkwater, Sir O. Richardson & W.E. Williams

of King's College London P.R.S. A 174 (1940) p. 164–188

used an reflexion echelon grating whereas previous work used a Fabry–Perot interferometer.

They conclude some support for Pasternack's suggestion but "we conclude that no real evidence has yet been obtained to show that the fine structures depart substantially from the values calculated from Dirac's equations."

Lamb. W.E. Jr and Retherford R.C.
P.R. 79 (1950) 549–572.
"Fine structure of the Hydrogen Atom" Part I
gives background to their 1947 experiment
First in a series of 6 papers on precision
measurement – culminated in Triebwasser,
Dayhoff & Lamb (1953)
Refer to early spectroscopic evidence, refer
to Kopfs series (1948) for spectroscopic
confirmation of level shift.
Microwave experiment suggested in 1928
by Grotrian. Attempts made in Germany
1932–1935 by Betz and Haase.
L & R used this work and applied
new microwave techniques developed
during the war.

~~Theory developed~~

Salpeter E.E. P.R. 89 (1953) p. 92–97
"The Lamb Shift for Hydrogen & Deuterium"
summarizes theoretical work up to 1953.

2 approximations 1) neglect coupling terms in α
2) approximate treatment of Coulomb field powers of αZ.

1st rel. calculation by Kroll, Lamb, French & Weisskopf. gave results ~~?~~ 6 Mc/s. too low.

then more accurate treatment of Coulomb field + 4th order (α^2) corrections to radiative coupling yield value $\frac{1}{2}$ Mc/s below Lamb's final result.

By 1960 two improvements
(1) revised 4th order anomalous moment
(Sommerfield, Petermann (1957)
(2) More accurate treatment Coulomb field by Layzer (1960) & ~~Fried~~ Fried and Yennie (1960) gave good agreement

But then new experiments by Robiscoe et al (Cosens) (1968) Robiscoe, Shyn (1970) who also revised Lamb's results, gave new expt. value near to 1057.9 instead of 1057.8 so old theory at 1057.56* (quoted by A Miller & Brodsky) was just too low.
This was broken in 1970 when

Applequist & Brodsky calculated α^2 terms and found disagreement with calculation of Soto (1966) and their new value was 1057.91 ± 0.16, so very close to experiment. Further refinements in period 1970 – 1972 increased precision of theory to the value quoted in L.P. de R. viz. $1057.911 \pm .012$, so theory now 5 times as accurate as the experiment.

Nobel Prize lectures 1965

Tomonaga explains his debt to Dirac's ~~many~~ multiple time formalism — extended to multiple times for fields (Dirac used particle description for electrons). Tomonaga says he began calculating Lamb shift after hearing about it "through the popular science column of a weekly U.S. magazine".

Feynman v. good account of early work with Wheeler on absorber theory of radiation → Lagrangian electrodynamics → ~~dug~~ space-time point of view.

Anecdote re Slotnick calculation of electron-neutron scattering. Feynman has checked out the problem with his method!! "The next day at the meeting I saw Slotnick and said 'Slotnick, I worked it out last night, I wanted to see if I got the same answers as

you do - - - - " and he said " what do you mean you worked it out last night, it took me six months" ~~and, he said,~~ " ~~what do you mean you worked it out last night, it took me six months!~~ "

And, when we compared the answers he looked at mine and he asked " what is q in there, that's q - -" I said " that's the 'momentum transferred by the electron' the electron deflected by different 'angles'". "Oh", he said " no, I only have the limiting value as q approaches zero; the forward scattering". Well it was easy enough to just substitute q equals zero in my form and I then got the same answer as he did. But, it took him six ~~years~~ months to do the case of zero momentum transfer whereas, during one evening I had done the finite and arbitrary momentum transfer . .

That was my moment of triumph in which I realized I really had succeeded in working out something worthwhile."

Feynman stresses heuristic value of reformulating theories in very different ways — "Perhaps a ~~they~~ thing is simple if you can describe it fully in several different ways without ~~knowing~~ immediately knowing that you are describing the same thing." Feynman says this is a powerful way of defining simplicity

"Theories of the known, which are described by different physical ideas may be equivalent in all their predictions and hence are scientifically indistinguishable. However they are not psychologically identical when trying to move from that base into the unknown. For different views suggest

different kinds of modifications which might be made and hence are not equivalent in the hypotheses one generates from them in ones attempt to understand what is not yet understood".

F. ascribes idea of positron as electron going backward in time to Wheeler — all electrons are 1 "knotted" particle (But as F. there are not equal nos. of + & – electrons).

F. remarks "the idea of the positron being a backward moving electron -- was very convenient but not strictly necessary for the theory because it is exactly equivalent to the negative energy sea point of view"...

Nafe, J.E, ~~Nelson~~ Nelson, E.B. & I.I. Rabi

P.R. 71 (1947) 914-915.

"The Hyperfine Structure of Atomic Hydrogen and Deuterium".

find discrepancy in theory, experiment.
5 times greater than probable error.

Breit, G. P.R. 72 (1947) p. 984.

"Does the Electron have an intrinsic magnetic moment" suggests that result can be interpreted in terms of an intrinsic magnetic moment (Pauli type)

Kusch, Foley P.R. 72 (1947) 1256.

give preliminary evidence of % from Zeeman splitting of atomic levels in Gallium.
these results written up in

Kusch, P and Foley H.M. P.R. 74 (1948) 250-263

" The Magnetic Moment of the electron.

while that Schwinger's calculation of the anomaly came after their measurement, but not because of it.
The experiment was due to Breit's suggestion of an anomalous moment — direct test using Zeeman splitting due to suggestion of Rabi

C.M. Sommerfield (1957) P.R. 107 p. 328–29.

"Magnetic Dipole moment of the electron"
reviews Peterman, Kroll calc. 4th order
to $\mu_s/\mu_0 = 1.0011596$.

A. Petermann Nuc. Phys. 5 (1958) p 677–683

"Fourth order Magnetic ~~Moment of the~~
Electron"
refers to Franken, Liebes measurement (1957)
as starting point of detailed calculation
of 4th order corr. They found
1.001165(11), need too high
for Karplus, Kroll.

Wilkinson D.T. & Crane H.R. P.R. 130(1963) p 852-86

"Precision Measurement of the g Factor of the Free Electron".

culmination of work initiated by Louisell, Pidd, Crane (1954), refined by Schupp, Pidd, Crane (1961) the latter experiment had errors same order of magnitude as 4th order corrections, new experiment devised to test 4th order prediction

W & C suggest not worth increasing accuracy of experiment to test α^3 terms (these terms contribute about 1.5 ppm perhaps to result. whereas their experiment to had about 27 ppm uncertainty. But uncertainty in value of α itself is ≈ ~~9 ppm~~ 5 ppm, similarly for other constants involved in evaluating the data.

Wilkinson, Crane's results were re analysed by Rich & others who found a three standard deviation from theory

New experiment undertaken by Wesley & Rich brought back agreement with theory.

Wesley J.C. & Rich A. P.R.A. 4 (1971) 1341 – 1363
"High-Field Electron g-2 Measurement".

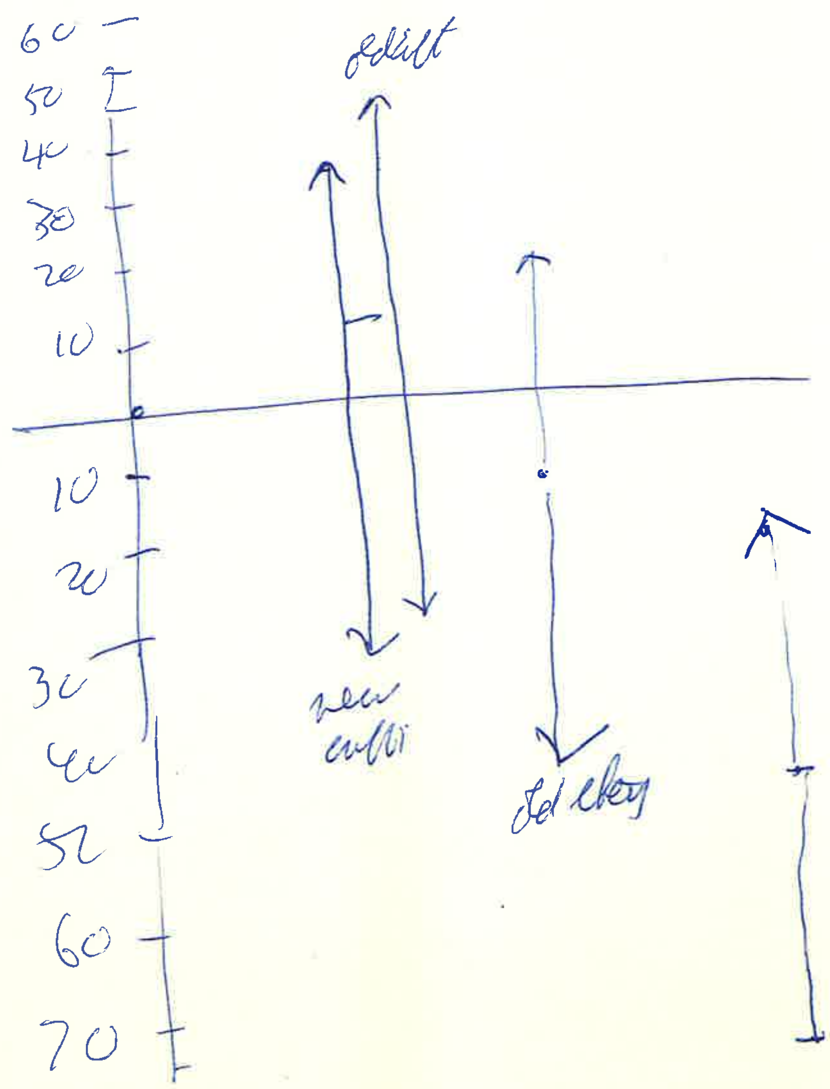
W & R increased accuracy to 3 ppm made possible a direct test of 6th order corrections. Two calculations being worked by 6 groups. Refer to Levine & Wright

W & R. say accuracy might be increased by a factor of 2, but "we feel that such an attempt would not be worth so large an investment of time & effort necessary. Accordingly we have concluded our current electron g-2 work".

W & R. admit discrepancy of their result with the revised C, W "no concrete basis for the discrepancy has yet been found".

Edit
W o e old   1 1 5 9 6 5 | 77 ± 35
reverd | 67 ± 35

Ulterij Ld P/de R. | 54 ± 33
2 o W. | 19 ± 25



Levine, M.J. & Wright, J. P.R.D 8 (1973) 3171–3179

Anomalous Magnetic Moment of the Electron

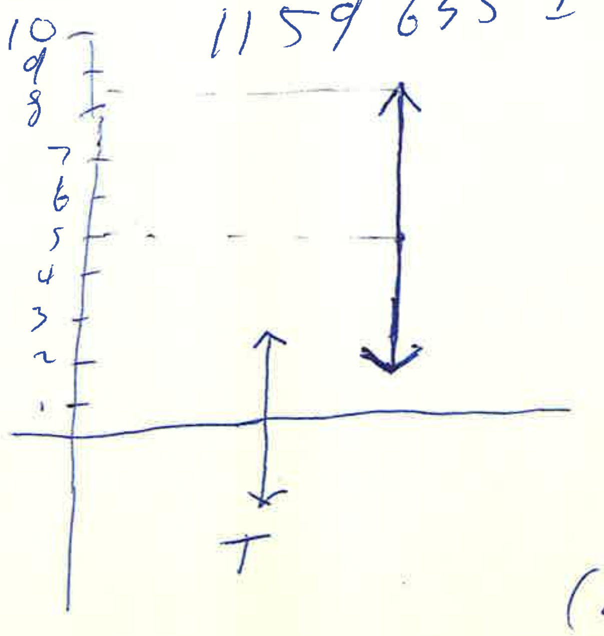
refer to Granger, ~~Ford~~ Ford (P.R.L. 28 (1972) 1479

for correction of W & R value from

1159657.7 to 1159656.7 ± 3.5

(i.e. L & W) they revise their previous value quoted by Wesley, Rich as being too high due to an error in numerical integration for one graph.

their final result 1159651.9 ± 2.5 compared with their previous value of 1159655 ± 2.



(10, 9 & F) they also correct Wilkinson, Crane's results & bring these into agreement with Wesley, Rich & with theory (acknowledged in Wesley, Rich R.M.P (1972) errata)

Combley F. and Picasso E:
"The Muon (g-2) Precession experiments: Past, Present and Future".
Physics Reports 14 C (1974) p 1-59.

---

refer to most recent calculations of of 6th order electron anomaly a_3.
Calmet and Peterman Phys. Lett. 47 B (1973) 369
Levine, Wright P.R. (1973)
Cvitanovic and Kinoshita SLAC Rep. (May 1974)
they quote theoretical value:
$a_\mu = .00 1165 897 \pm 8$ Bohr magneton

Expt. Bailey et al. (Nuovo Cimento A 9 (1972) p 369)
$a_\mu = 0.00116\ 6160 \pm 310$

Bailey et al. Phys. Lett. 55 B (1975) p. 420–424.

New measurement of (g-2) of the muon.

Experimental value $\frac{g-2}{2}$ = .00 1165895 ± 27

theory = 0.00 1165908 ± 10

(includes 73 ± 10 from hadrons))

Cvitanovic, P & Kinoshita T.

P.R. D 10 (1974) 4007– 4031

Sixth order magnetic moment of the electron

Obtain a theor. = .00 1159651.7 ± 2.2

experiment = .00 1159656.7 ± 3.5

$$(\text{cf } P(\chi^2_{x-y}) = \int e^{-\frac{(x-x_0)^2}{\sigma} - \frac{(y-y_0)^2}{\sigma'}} \, d\cancel{x} \quad \text{with } y = x + \Omega.$$

$$= \int e^{-\frac{(x-x_0)^2}{\sigma} - \frac{(x+\Omega-y_0)^2}{\sigma^b}} \, dx.$$

$$= \int e^{-\frac{2x^2}{\sigma} + \frac{\Omega^2}{\sigma} + \frac{x_0^2 - y_0^2}{\sigma} + 2x(x_0+y_0) + 2x\Omega - 2y_0\Omega}.$$

This is most recent, probably most accurate value. to date.

J. Calmet, S. Narison, M. Perrottet and E. de Raphael. "The anomalous magnetic moment of the muon: A Review of the theoretical calculations". RMP. 49 (1977) 21–29.

α_μ (theory) = 0.00 1165 920.6 ± 12.9
α_μ (expt.) = 0.00 11 65 895 ± 27
(Bailey et al 1975)

for Comparison
α_e (theory) = 0.00 1 159 652.4 ± 0.6
α_e (expt) = 0.00 1 159 656.7 ± 3.5
Wesley, Rich

Refer to
Levine, M.J. Perisho, R.C and Roskies, R. (1976) Phys. Rev. D. 13, 997–

~~R.S. Van Dyck Jr, P.B. Schwinberg, H.G. Dehmelt~~
"Precise Measurements of ~~Axial, Magnetron,~~ Cyclotron and Spin-Cyclotron-Beat Frequencies on an Isolated 1-MeV Electron". PRL 38 (1977) P. 310–314
quote α_e (expt.) = 0.00 1159652.4 ± 0.2

R. Haag. Dan. Mat. ~~M~~ Fys. Medd. 29 no 12 (1955), 1–37.
"On Quantum Field Theories".
discusses difficulties connected with ∞ no degrees of freedom – shows free field vacuum of Tomonaga-Dancoff and Dyson's $U(t_1, t_2)$ for finite t_1 or t_2 does not exist. Shows general field theories exist (i.e. commutation relations for equal times are not fixed a priori) which are just as wide as rule as S-matrix theories. Proves in §2 every S-matrix can be derived from some field theory – In §3 discusses extra restrictions if field operators commute at equal times → $U(t_1, t_2)$ cannot exist for those theories for finite t_1 or t_2. Mathematically this is due to existence of inequivalent representations of the commutation relations in non-separable Hilbert spaces (continuum of orthonormal basis vectors) because $\aleph_0^{\aleph_0}$ for infinite no of oscillators and each infinite no of states.

Hall, D and Wightman, A. S.

Mat. Fys. Medd. Dan. Vid. Selsk. 31 (no 5) (1957) 1–39.

"A theorem on invariant analytic functions with applications to Relativistic Quantum Field Theory".

Mainly concerned with a purely mathematical result

"An analytic function of n four vector variables invariant under the orthochronous Lorentz group is an analytic function of their scalar products".

This theorem was used earlier first in

Wightman (1956)

He suggests

Wightman, A.S. and Schweber, S.S.

P.R. 98 (1955) 812 – 837

"Configuration Space Methods in Relativistic Quantum Field Theory".

They discuss representations of commutation relations for finite nos. of particles refer to Jordan, Wigner's proof of equivalence of all reps. For ∞ nos of states we have to assume also unique vacuum, but this may be inconsistent with eqns of motion. (cf Van Hove (1952)).

Wightman, A.S. P.R. 101 (1956) 860 – 866

Quantum Field Theory in terms of vacuum expectation values

introduces the Wightman-axioms for VEV's. Shows VEV's are boundary values of analytic functions, and that field theory can be 'recovered' from properties of VEVS.

Lehmann, H, Symanzik K and Zimmermann W.

N.C. 1 (1955) 205 – 225.

"Zur Formulierung quantisierter Feldtheorien"

introduce LSZ formulation — ~~the~~ eqns contain no renormalization constants, involve only experimental masses & couplings. Advantage is "elimination of all divergent terms in the basic equations". In power-series expansion we recover "renormalized expansions of the conventional formalism".

L.S.Z. N.C. 6 (1957) 319–333

"On the Formulation of Quantized Field Theories – II"

refer to form of S-Matrix approach

Heisenberg Z f Phys. 120, 513, 673 (1943).

discuss derivation of dispersion relations

– refer to Goldberger P.R. 99 979 (1955) and Oehme R.A. N.C. 4 1316 (1956), Symanzik, P.R. 105, 743 (1957)

for the development of the LSZ formalism.

Greenberg O. W. "Haag's theorem & clothed operators"
P.R. 115 (1959) 706–710.
discusses more complete proof of Haag's theorem.

Matthews P.T. and Salam, A
P.R. 94 (1954) 185–191
"Renormalization".
All divergences (charge & mass) are dealt with by
counter terms so that theory is now finite
in all orders of perturbation i.e. charge renormalization
is dealt with in same way as mass renormalization,
following ideas of Gupta. Proc. Phys Soc. A 64 426 (1951)
by introducing a counter term.

Lehmann, H. N.C. 11 (1954) 342 — 357
"Über Eigenschaften von Ausbreitungsfunktionen und Renormierungskonstanten quantisierter Felder

introduces spectral representation for Δ_F' and S_F', and discusses magnitudes of Z_2 and Z_3, assuming all functions exist.

Was a development of approach of Källén, G. Helv. Phys Acta 25 (1952) 417 - 434. "On the definition of the renormalization constants in quantum electrodynamics" who formulates a field theory in terms of renormalized operators, and expresses renormalization constants in terms of renormalized fields.

Källén raises two problems:

1) Is Dyson series convergent, i.e. are renormalized quantities themselves finite (i.e. exist) and are not just a divergent sum of finite terms.

2.) In a sensible theory, i.e. coupled, the Z's are

well defined and finite. Källén does not discuss whether this is true in actual example, but it is quite a separate question from no 1.

Stueckelberg E.C.G. and Petermann, A
Helv. Phys Acta 26 (1953) 499–520
"La Normalisation des constantes dans la theorie des quanta")
Introduced ideas of a renormalization group.

Gell-Mann, M and Low F.E.
"Quantum Electrodynamics at small distances".
Phys Rev 95 (1954) 1300
refers to Källén Dan. Fys Medd 27, 12 (1953) 1–18.
"On the Magnitude of the Renormalization constants in quantum Electrodynamics"
e_0 may be infinite or finite (unlikely) de, claims one of Z_2^{-1} e_0^2 and m_0 must be infinite.
Källén says renormalization constants may

actually be finite - they only appear infinite because relevant functions are not analytic at $e \to 0$.

(cf $\exp(-e^2/L) \simeq 1 - \frac{e^2}{L} + \ldots$ where $L \to 0$.

each term diverges.

or $\exp(-e^2 L) \simeq 1 - e^2 L + \ldots$

but exponent regular at ∞

i.e. renormalization constants may be finite even if expansion is infinite.

or renormalized series may be ~~infinite~~ divergent even if terms are finite e.g. e^{-1/e^2} has no convergent power series expansion around $e = 0$.)

But Kallen argues that one renormalization constant is actually infinite in Q.E.D

The best we can hope for is that the renormalized theory is finite

- he does not discuss this point in detail.

N.B. this argument for infinite value of at least one renormalization constant has rigorously been criticized.

Jaffé, A. Comm. Math. Phys. 1 (1965) 127–149.

"Divergence of Perturbation theory for bosons"

investigates convergence of field theory, 2 2-dimensional space-time — finds Green's functions not analytic at $\lambda = 0$. For non-derivative couplings these theories are renormalizable and all renormalizations are finite.

~~Jaffé find~~ refers to work of Hurst (1952) Proc Roy Soc. 214 A 44 and Thirring (1953) & Petermann (1953) for for $\lambda\phi^3$ theory in 4-dimensional space. time for proof of non-analyticity in DF.

Hepp, Klaus. Comm. Math Phys. 2 (1966) 301–326

"Proof of ~~the~~ Bogoliubov–Parasiuk theorem (1957) on renormalization". is very ~~technical~~ paper. Here Salam's criterion is not satisfied by Bogoliubov "It is hard to find two theoreticians where understanding of the essential steps of the proofs isomorphic".

W. Thirring

"On the divergence of perturbation theory for quantized fields.

Helv. Phys. Acta. 26 (1953) 33–52.

Refers to Dyson's physical argument.
Considers $\lambda\phi^3$ scalar theory – shows this is renormalizable – estimates lower bound for nth order pert. multiplier & estimate for these graphs finds sum behaves like

$$\lambda^n \frac{(n/2-2)!}{n^2} \times \text{const.}$$

diverges for all values of λ

$$\frac{R_{n+1}}{\lambda^n} = \lambda \cdot (n+1) \times \left(\frac{n}{n+1}\right)^2 > 1 \text{ as } n \to \infty.$$

So roughly each graph $> \lambda^n/n^2$

no. of graphs $\sim n!$

$\therefore$ nth term is like $\lambda^n \cdot \frac{n!}{n^2}$

Hurst C.A. Proc. Camb. Phil. Soc. 48 (1952) 625–639. also derives divergence of $\lambda\phi^3$ field theories

theory also suggests power series in (λ-λc) also diverges so also non-analytic for all values of coupling constant λ.

Petermann Helv. Phys. Acta. 26 (1953) 291 also reached similar conclusion.

Hurst ~~A.E.~~ C.A. Proc. Roy Soc. 214 A (1952) 254–61.

"The enumeration of graphs in the Feynman-Dyson technique"

investigates no of graphs of order n, shows rapid increase, suggests perturbation series cannot converge unless contribution of each graph goes down very rapidly with n.

concludes no of graphs is of order $n^{n/2}$ asymptotically. Hurst concludes

"excellent agreement between experimental results and theoretical calculations would indicate that the series is in fact to be interpreted as an asymptotic expansion about the singular point $\lambda = 0$"

P.J. Redmond . P.R. 112 (1958) 1404–1408
"Elimination of ghosts in propagators"
suggest D_F' has an essential singularity at $q^2 = 0$. He concludes Z^{-1} is finite. How then if we expand Z^{-1} in power of q^2 do we find a series all of whose coefficients are infinite?
Redmond says $Z^{-1}(q^2)$ is not analytic at $q^2 = 0$.

Uretsky J.L and Redmond P.J.

Ann. of Phys. 9 (1960) 106–124.
"In-out-formalism for nonrelativistic quantum theories" calculates N.R. model treated à la LSZ with known properties of N.R. models – throws some light on LSZ in rel. theories – do not discuss essential singularities.

Hori S. Prog. Theor. Phys. 8 (1952) 569–570.
(Letter) "On the Convergence of the S-Matrix Series"

refers to Dyson (1952) and evidently confirms Hurst's estimate for no. of graphs, but does not estimate lower bound for contribution from each graph, as e.g. in Thirring's work.

R. J. Riddell "The Number of Feynman Diagrams"

P.R. 91 (1953) 1243–1248.

Riddell confirms Hurst's estimate for no. of relevant graphs ~ $n^{n/2}$. Concludes series is not absolutely convergent. "Size of coupling constant seems to have little to do with the convergence of the theory (although in an asymptotic expansion the size would determine the usefulness of the theory)" Could nth order term of series be conditionally convergent due to cancellations between terms (of a fixed order n) – very difficult point to investigate. If series is conditionally

convergent "Padé type approximations based on rearrangements of the series seem of doubtful validity."

Dyson F.J. P.R. 85 (1952) 631–632.

"Divergence of Perturbation Theory in Quantum Electrodynamics"

Heuristic argument for divergence of $F(e^2) = a_0 + a_2 e^2 + a_4 e^4 + \dots$ for any physical quantity F

Take $e \to ie$. If F is absolutely convergent so $F(-e^2)$ but this does cannot be expected to converge (correspond to a well-defined function") since we are now in a world where like opposite charges repel. So we can get an "explosive disintegration of the vacuum by spontaneous polarization".

Potential energy of pairs can be greater than rest energy in region. $\frac{(ne)^2}{a} > n m c^2$

i.e. $n > \frac{mc^2}{e^2} a$. Take $a = \hbar/mc$ then $n > \frac{\hbar c}{e^2} \approx 137$.

Dyson argues that terms will decrease and then increase without limit at the critical value of $n \approx 137$.

Dyson goes on to argue. There are 2 alternatives

A $F(e^2)$ is well-defined by non-perturbative analytic formalism with asymptotic expansion $F(e^2) = a_0 + a_2 e^2 + \ldots$ but the series is not sufficient by itself to define $F(e^2)$ uniquely

B Formalism only gives us coefficients a_0, a_2 etc and hence formalism does not determine $F(e^2)$ — so new physical theory is needed to fix $F(e^2)$ — Dyson's says B has attractive features — would account success of Q.E.D. with need to extend theory to deal with meson phenomena etc.

Hurst Proc Camb Phil Soc. 48 (1952) 625-639 cont'd.
"A divergent Perturbation Expansion in field theory"

quote" Error in stopping calculation at any order of approximation is of smaller order than last term calculated.

$z=0$ cannot be a regular point for an asymptotic expansion but would be an essential singularity, which may or may not be isolated. In this case the asymptotic expansion would be valid only in a sector $|\arg z| < \theta$, $\theta \leq \pi$ and error committed by stopping at some order of approximation would not be capable of indefinite reduction by calculating further terms." The difference between an asymptotic and a convergent series has more in the theoretical than in the practical consequences, for large complex arguments would lead to a series which effectively ~~orders~~ orders defined for calculations whose sizes are "asymptotically or as convergent."

"For a convergent expansion i.e. z in the field, the error could be made as small as one pleased by increasing n, but for an asymptotic expansion the error would only be decreased to a non-zero lower limit, and then computing further terms, the error would begin to rise again".

Low, F.E. P.R. 97 (1955) ~~pp~~ 1392 – 1398

"Boson-Fermion Scattering in the Heisenberg Representation"

derives the Low equations by direct manipulation

Chew G.F. and Low F.E. P.R. 101 (1956) 1570 – 1579

"Effective-range approach to the Low-Energy p-wave Pion-Nucleon Interaction"

Rederives Low eqn using Wick's method (R.M.P. 27 339 (1955)) and applies to obtain effective range approximation for P-wave phase shift.

Essentially C & L have a fixed t dispersion relation which they approximate by neglecting all but P-waves. Resulting equation is not a partial wave dispersion relation for the P-waves which allows for coupling in of all higher partial waves.

Oehme R. P.R. 100 (1955) 1503 – 1512

"Dispersion Relations for Pion-Nucleon Scattering.

I. The Spin-Flip Amplitude"

consider dispersion relations off the forward direction (actually for derivatives w.r.t. t at forward direction).

and ~~we~~ then the unitarity equation is just Low's equation. If we neglect contribution from phase shifts $\gtrsim 1$.
Low's equation, no crossed, because Oehme claims "there do not exist dispersion relations for amplitudes corresponding to undisturbed angular momenta".

Oehme R. P.R. 102 (1956) 1174 – 1180
"Dispersion relations for Pion-Nucleon scattering: No-Spin-Flip Amplitude".

Again Oehme first derives dispersion relation – then makes 3 assumptions neglect recoil of nucleon, neglect ang. momentum states > 1, neglect inelastic processes.
– and arrives at Low's equation.
In final section of paper Oehme remarks he has derived approximate equations for undisturbed angular momentum amplitudes but says he cannot find exact dispersion relations for undisturbed angular momenta

— in effect he has not put in enough information except in approximation of simply neglecting any "interaction" with the higher angular-momentum states.

<u>Chew G.F., Goldberger, M.L., Low, F.E. and Nambu, Y.</u>

P.R. <u>106</u> (1957) 1337 — 1344

"Application of Dispersion Relations to Low-Energy Meson-Nucleon Scattering"

Describe 2 uses of dispersion relations

1.) Experimental check of forward dispersion relations by measurement of [illegible] quantities
2.) Neglecting higher waves than $l=1$, derive Chew-Low equations as an [illegible] of [illegible] but in static limit.

CGLN unable to affect the validity of the static limit, but CGLN <u>allow for the effect of recoil</u>.

Mandelstam, S. P.R. 112 (1958) 1344 – 1360.

Suggests that dispersion relations could reflect eqn of field theory ascribed to Gell Mann at Rochester Conference in (1956.

Mandelstam introduces his double dispersion representation "the simplest assumption we could make is that it [the amplitude] is analytic in the entire space of the two variables [s, θ] except for cuts along certain hyperplanes" [determined from unitarity]. Assumption is a generalization of a representation due to Nambu (1955), but Nambu's rep. is invalid in perturbation theory, whereas M's rep. for representation "has not been shown to be invalid" – quotes examples of perturbation terms which do have this representation.

He then explains that Mandelstam iteration as a practical proposal. Title of paper is "Determination of the Pion-Nucleon Scattering Amplitude from dispersion relations and Unitarity. General Theory". M. argues ~~one~~ singularities that arise in all orders of perturbation theory should be expected to be present in full amplitude if latter contains all singularities forced by unitarity.

△

Mandelstam S. Rep. on Prog. in Physics 25 (1962) 99–162.
"Dispersion relations in Strong-Coupling Physics".
general review article.
Discusses CGLN approach of including just a few partial waves in fixed-t dispersion relations. Contrasts phenomenological application and dynamical application – how much information is taken from experiment – two approaches shade towards one another if arbitrary parameters introduced to approximate unknown effects in the dispersion relation which are then used to fit experiment. However "in practice one finds a difference between the types of calculation which characterize the two approaches." (p122)
Mandelstam formulates principle of maximum analyticity thus "the scattering amplitude is analytic in all its variables except at those points where singularities arise as a consequence of the unitarity condition" (p134)
on p.158 "my approach is from point of view of local field theory" He attacks pure S-matrix approach. "though quantum field theory may

contain operators which do not correspond to measurements, the S-Matrix theory seems to go too far in the opposite direction and to contain less than could actually be measured".

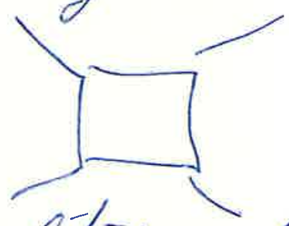
- . . "Another objection is that assumptions about the possibility of analytically continuing our functions are rather artificial unless they are deduced as mathematical consequences of some other postulates". Our present quantum field theory, ~~and~~ in spite of its inadequacies, is based on well-worn concepts of classical field theory and quantum mechanics and would appear to be more natural. This, needless to say, is a matter of opinion about which it is difficult to argue and, in any case, one may maintain that such considerations should play no part in formulating a theory. One must keep an eye on all possibilities with regard to making a radical extension of the theory which all agree is necessary".

P. 29 there will "always be an uncertainty whether disagreements [with experiment] were due to the failure of the approximations or of the dispersion relations themselves"

<u>Mandelstam S.</u> P.R. <u>115</u> (1959) 1741–1751.

" Analytic Properties of Transition Amplitudes in Perturbation Theory".

Says double dispersion relations "have not yet been proved from the general principles of quantum field theory". 4th order perturbation term



are investigated and shown to have the required representation provided the masses do not allow anomalous thresholds. In general anomalous thresholds (of the second type in classification of Karplus, Sommerfield & Wichmann (P.R. <u>114</u> 376 (1959)) implies "the double dispersion representation breaks down, and there will be singularities in the complex plane".

~~Goldberger M.L.~~ <u>Gell-Mann, M. Goldberger M.L. Thirring W.E.</u>
P.R. <u>95</u> (1954) 1612 — 1627.

"Use of Causality Conditions in Quantum Theory".

refers to link between causality and scattering amplitude relations going back to Kramers (1927) and Kronig (1926) [J. Opt. Soc. Am. <u>12</u> 547 (1926)] also ~~Kram~~ Kronig, Physica <u>12</u> 543 (1946) suggested causality requirement should be imposed, in addition to Lorentz invariance and unitarity, on the S-matrix.

G G-M and T deduce forward scattering dispersion relation for photons by nuclei. They use perturbation theory to estimate required analytical properties.

<u>Goldberger</u> M.L. P.R <u>97</u> (1955) 508 — 510

derives exact dispersion relation, not using perturbation theory for the case of photon scattering in forward direction. Title is "Use of Causality conditions in Quantum Theory"

Goldberger M.L. P.R. 99 (1955) 979–985.
"Causality Conditions and Dispersion Relations. I. Boson Fields"

treats case of forward scattering of bosons
with finite rest mass

Toll J.S. P.R. 104 (1956) 1760–1770.
"Causality and the Dispersion Relation: Logical Foundations".

derives dispersion relations for general causal linear systems with time-independent connection between the output and a freely variable input. refers to applications in optics to Wolf & Herzfeld Handbuch der Physik (1928) vol. 20 ch 10 p 480, Kronig & Kramers RMP 4 471 (1932) for applications in electrical network theory. Toll gives simple physical picture of dispersion relation. Scattering amplitude ~~proc~~ proceeds as contributes to coherent scattering amplitude as 2 stage process i.e. absorption + re-emission

Imag. coherent amplitude relates to real absorption
= absorption amp × re-emission amplitude

→ Im forward scattering = Total cross-section.

Real coh. amplitude relates to virtual absorption
and is expressed as sum of (emission × absorption)
over all intermediate states
this is just the dispersion relation.

Ravenhall? S. N.C. 15 (1960) 658 – 685.

"Some Rigorous Analytic Properties of Transition Amplitudes"
discusses double dispersion relations from
first l ... of general field theory
– from analytic in a certain domain
– no general proof of the Mandelstam
representation achieved

<u>Mandelstam</u> S. Ann. Phys. <u>21</u> (1963) 302–343

"Regge Poles as Consequences of Analyticity and Unitarity

M. discusses <u>proof of analyticity in J</u> if double dispersion relation is satisfied — sort of reverse of Regge's original programme.

He states on p. 313 that ~~to prove~~ the necessary absence of C-M- ambiguity.

It does <u>not</u> discuss equivalence of removal of C.M. ambiguity and assumed analyticity of 2nd Regge specifically, but his derivation appears to imply this.

Donnachie, A, Hamilton J and Lea A-T.
P.R. B135 (1964). 515–539.
"Prediction of p-, d- and f- wave Pion-Nucleon Scattering

use partial wave dispersion relations to get π-N phase shifts with input of experimental values of resonances in crossed channels (so this is not a bootstrap situation)

Donnachie A & Hamilton J. Ann. Phys. 31 (1965) 410–435
"The Quantum numbers of the Nucleon Isobars".

continues the DHL programme with analogs of $N, N^*, (\pi\pi)_0, \rho$ as principal contributions.

Hamilton J and Woolcock, W. S. R.M.P. 35 (1963) 737–787
"Determination of Pion-Nucleon Parameters and phase shifts by Dispersion Relations".
Extend CGLN to determine coupling constants etc (i.e. they are fixed by dispersion relations)

H & W state main limitation is convergence of inverse of Legendre Series. (cf CGLN)

Donnachie, A. Rep. Progress in Physics (1973) 36 695–753.
"Partial Wave Analysis and Baryon Resonances".
General phenomenological review, no detailed dynamical calculations.

Chew G.F. and Mandelstam S. P.R. 119 (1960) 467–477
"Theory of the Low-Energy Pion-Pion Interaction".
Derives partial wave dispersion relations derived from Mandelstam representation. Solutions N/D method of soln. They refer to C.D.D. ambiguity "They correspond to theories in which, before the coupling is turned on, there are one or more particles with the same quantum no's as two pions. Once the coupling is turned on, these

particles become unstable, and appear experimentally as resonances. These "kinematical" resonances differ from "dynamical" resonances . . . in that they occur for arbitrarily small values of the coupling constant. The absence of such unstable particles must be regarded as an additional postulate to be inserted into the theory".

Chew G.F., Mandelstam S. and Noyes H.P.

P.R. 119 (1960) 478 – 481.

"S-wave dominant solutions of the Pion-Pion Integral equations". continues the Chew & Mandelstam paper (1960) and introduces the phrase bootstrap

"This would be a "bootstrap mechanism"; i.e. the force producing the P-wave resonance would be due to the exchange of a resonating P-wave pion pair". In effect the P-resonance is being bootstrapped.

Chew G.F. & Frautschi S.C. P.R.L. 7 (1961) 394-397.

"Principle of Equivalence for All Strongly Interacting Particles within the S-Matrix Framework".

They refer to C.D.D. ambiguity but claim some C.D.D. poles may not correspond to elementary particles. They start from Regge theory. Poles position $\alpha_i(E)$ is analytic function of E. If "$\alpha_i(E)$ = const." ~~or not allowed~~ turns out to be not allowed. They refer to Feynman for idea that correct theory should not allow a decision as to which particles are elementary. They refer also to Heisenberg (RMP 29 269 (1957)) for idea of "an underlying spinor field that corresponds to no particular particle but which is supposed to generate all the observed particles on an equivalent bases". C & F postulate "all poles of the S matrix are Regge poles." i.e. moving poles in the j-plane. "This is a natural

extension to angular momentum of of the maximal analyticity Principle which hitherto has been applied only to linear momenta"
"Maximal analyticity in linear momenta fails to specify precisely the asymptotic behaviour in momentum transfer, which is the controlling factor in determining the region of analyticity in the J plane" (see Froissart)

Experimental test of hypothesis to place all particles on Regge trajectories connected with high energy scattering cross-sections in crossed channels.

Elementary particles give asymptotic behaviour $\alpha(t) \to$ S = const. 10.
the difference should show up experimentally (slope of peak does not change with energy for example).
Elementary particles with low mass particles have low spins.

Regge T. N.C. 14 (1959) 951–976

"Introduction to complex Angular Momenta".

Introduced complex j in N.R. scattering theory — increases range of validity of dispersion relations — up to on double Mandelstam representation.

(Mandelstam rep. also derived for potential scattering independently by Blankenbecler, Goldberger, Khuri and Treiman, Ann Phys. 10 (1960) 62–93)

Zachariasen F. and Zemach C. P.R. 128 (1962) 849–858

"Pion Resonances".

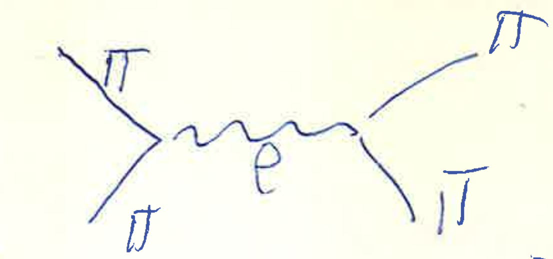
derives the ρ-bootstrap

ρ exchange provides attractive force between 2 pions to produce the ρ resonance.

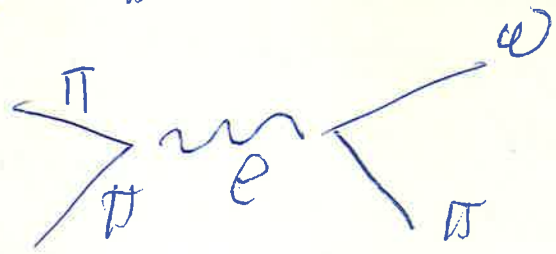
Full bootstrap v. difficult. e.g. we say that π-mass as given in $\rho = \pi\pi(\rho)$

Also we should treat Regge families not just single particles. In simple approximation we ignore the Regge behaviour. (replace by phenomenological cutoff for example to reproduce effect of Regge behaviour in making amplitudes less singular at large energies)

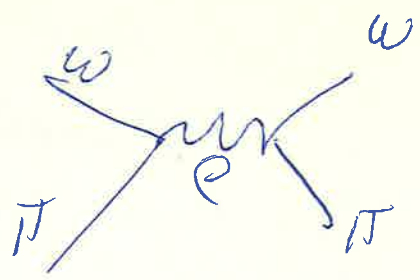
Refer to early calculation of Chew & Mandelstam (1960). Z and Z now consider also $\pi\pi \to \pi\omega$ and $\pi\pi \to K\bar{K}$. they ignore the $K\bar{K}$ channel but include the effect of the ω. Effect of ω very important. In Z & Z consider only ρ exchange but include the effect of the $\rho\omega$ channel



and



and



We find $m_\rho \approx$ 654 MeV.
cf with expt. value 767 MeV.
needed for widths.

without the ω channel
We find $m_\rho =$ 350 MeV
taking $\rho = \pi\pi(\rho)$ alone

We also consider the problem of the ω as a resonance in $\rho\pi$ scattering. We see a certain that results are fairly insensitive to its exact magnitude.

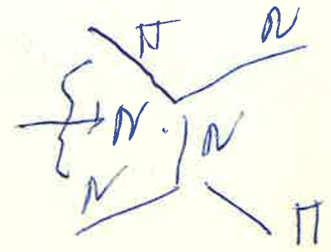
Abers, E. and Zemach C. P.R. 131 (1963) 2305–2318.

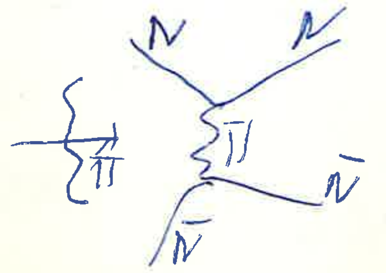
"Bootstraps and the Pion-Nucleon System".

Good general introduction to the bootstrap philosophy:

In bootstrap forces are, in fact, so strong that the binding energy of the "composite" nucleon $[(N+\pi)]$ is comparable to the masses of the constituents. This is the essential feature that permits a particle to appear as a constituent in a composite state and as the composite state itself thus obscuring (though not invalidating) the analogy to loosely bound states such as the hydrogen atom or an atomic nucleus.

Simplest case of a bootstrap resonance would be $N = \pi N (N)$ and $\pi = N\bar{N}(\pi)$





But N exchange is too weak to bind πN system.

Also π is very deeply bound state of $N\bar{N}$ we should start with $\pi = \pi\rho$ and $\pi = K\bar{K}$ as dominant effects.

A, 2 include effects of N and N^* exchange and also ρ exchange in crossed channel $\pi + \pi\ (\rho) = N + \bar{N}$.

Results for masses, widths of Λ, N^* depend on cutoff's — reasonable value for cutoff's (due to neglect of high energy behaviour) could give agreement with experimental values.

2, 2 refer to Chew (P.R.L. 9, 233 (1962)) for physical suggestion of the "reciprocal bootstrap" mechanism.

Kramers H.A. Atti Congr. Intern. Fisici, Como, 2 (1927) 545–557.

"La Diffusion de la lumière par les atomes".

K. derives connection between refractive index and absorption coefficient for scattering of light in the form of a dispersion relation – says he worked this out in an unpublished paper in 1925. Refers to independent discovery by Kronig in 1926 (J. Opt. Soc. Am. 12 547 (1926)).

~~Krame~~ Kronig R. de L. Physica 12 (1946) 543–544

In a letter to editor entitled "A Supplementary Condition in Heisenberg's theory of elementary particles".

refers to H's discussion of S-matrix – what features in of a future theory must be retained – refers to Kramers & Kronig relation for scattering of light by atoms from causality. "It hence would seem reasonable to

postulate for the scattering of particles a connection between the real and imaginary part of the scattering amplitudes of the same type as in optics."

J. Hamilton Prog. in Nuclear Phys. 8 (1960) 143-194.
"Dispersion Relations for Elementary Particles"

H. gives very good general review of dispersion relations in classical physics. e.g. Complex refractive index $\Omega(\omega) = 1/n(\omega)$

then $\mathrm{Re}\,\Omega(\omega) = \frac{2}{\pi} P \int_0^\infty \frac{\omega' \,\mathrm{Im}\,\Omega(\omega')}{\omega'^2 - \omega^2} d\omega'$

$\mathrm{Im}\,\Omega(\omega) = -\frac{2\omega}{\pi} P \int_0^\infty \frac{\mathrm{Re}\,\Omega(\omega')}{\omega'^2 - \omega^2} d\omega'$

derived with these formulae for $\Omega(\omega)$ by circuit analysis, and by general argument of causality. Use crossing relation $\Omega(\omega) = \Omega^*(-\omega)$ to change from $\int_{-\infty}^{\infty}$ to $\int_0^{\infty}$ in dispersion relation.

Sum rules obtained by letting $\omega \to \infty$ in dispersion relation,

H. also derives scattering of light by an atom by detailed model and by causality argument.

H next derives applications in particle physics — esp. M. representation is "not derived from causality but is deduced from a very general hypothesis about the analytic properties of scattering amplitudes when not only the energy but also other arguments variables (such as the momentum transfer) become complex".

H derives however a Thal con-relations at all energies we can compute the forward scattering amplitude and compare with experiment — good agreement occurred in other phenomenological applications.

H. discusses problem of proving more general dispersion relations " It may be that in order to prove the relation for larger values of the momentum transfer, we shall have

to we need more detailed knowledge
about the interaction"

~~Haag~~ Brenig W & Haag R. Fortschritte der Physik
7 (1959) 183–242.

" Allgemeine Quantentheorie der Stossprozesse "

---

gives general relativistic discussion of scattering theory in German.

W. Heisenberg Z. f. Phys. 120 (1943) 513–538.

"Die "beobachtbaren Grössen" in der Theorie der Elementarteilchen."

Part II of this paper is same vol. pp. 673–702.

H. discusses fundamental topics and says he is looking for features which will remain valid in a future theory (cf. perhaps Einstein's approach to special relativity), and lists the general features of the present theory which will survive in any future theoretical development.

Kronig's (1946) comment on H cont'd

"Heisenberg has investigated the problem in what respect the quantum theory in its present form will have to be modified in future and which of its features ~~will~~ certainly will have to be retained thus determining what might be called upper and lower limits between which the ultimate theory must lie.

--- It is the object of ~~this~~ note to point to a further restriction which it seems reasonable to impose on any future theory" .. criterion with the idea of extending dispersion relations as a condition on the scattering matrix. (directly but expressing indirectly a principle of causality which he takes to be the significance of dispersion relations in classical physics).

R. J. Eden "Regge Poles & Elementary Particles"
Rep. Prog. Phys. (1971) 34 995–1053.

Gives very good review of Regge theory, concludes with introduction to F.E.S.R, duality and the Veneziano representation.

Bailin D "The theory of weak interactions in Particle Physics"
Rep. Prog. Phys. (1972) 34 491–599.

Very good general review of weak interaction physics — good section on T-violation in $K_L \to 2\pi$ as well as 3π — decays possibility of a superweak T-breaking interaction. Also in the 1964 Fitch & Cronin experiment.

~~Stapp H.P. P.R. D 3 (1971) 1303 — 1320.~~

"S-matrix interpretation of quantum theory"

Stapp stresses ~~S-matrix makes all~~ predictions that are possible in quantum theory. We must not "interlock" too strongly with observed system for observation to make sense. S. discusses Bell's theorem and discusses bootstrap philosophy a "web" philosophy as Stapp calls it. He draws analogy with Whitehead Process & reality.

S refers to link between analytic properties assumed in S-matrix theory and the principle of macro causality which says "Every reaction is transferred over macroscopic distances only by physical particles; any transfer of energy-momentum not ascribable to net-work of physical particles has a probability that falls off exponentially under space-time dilation".

Stapp recommends a "pragmatic" attitude towards philosophy of QM — espoused in S-matrix approach.

S claims analyticity → macro causality but not micro causality

Equivalence between analytic structure and macrocausality is proved in

D. Iagolnitzer & H. P. Stapp.
Comm. Math. Phys. 14, 15–55 (1969)

"Macroscopic Causality and Physical Region Analyticity in S-Matrix Theory".

Equivalence is two-way.

Thus S-mat. general analyticity may not be derivable from field theory and with macro causality — macro and micro causality are two quite separate principles. Only macro causality can be used to justify the normal analytic structure in the physical region.

We require in addition a fourth of general analyticity to extend these primitive domains of analyticity.

Stapp says it is still an open question whether "normal analytic structure is at all consistent with unitarity" — an extra assumption such as macro causality may be needed.

D. Iagolnitzer : The S Matrix, (Amsterdam: North Holland) 1978

link between analyticity in physical region & macrocausality derived in Iag. & Stapp Comm. Math Phys 14 (1969) 15, based on Chandler & Stapp J Math Phys 10 826 (1969)

Wanders N.C. 14 (1959) 168.
Omnes P.R. 146 (1966) 1123

1st treatment of macrocausality.

I. contrasts traditional heuristic approach to analyticity due to
Gunson J. Math Phys. 6 827, 845 (1965)
derives CPT & spin-statistics theorem — Stapp. P.R. 125 (1962) 2139
Olive P.R. 135 B (1964) 745.

and Bloxham, Olive and Polkinghorne
J. Math. Phys. 10 (1969), 494, 545, 553.

I. says proofs here are incomplete and indeed incorrect due to absence of "[illegible] [illegible]

and mathematical tools 4. (p.245)

Gunson J. "Unitarity and On-Mass-Shell Analyticity as a Basis for S-Matrix Theories.
J. Math Phys. 6 (1965) I. p. 827–844
II. p. 845 – 851
III p. 852 – 858.

g. attempts to establish maximal analyticity as a consequence of unitarity + on-mass-shell analyticity.

Olive D.I. P.R. 135 B (1964) 745–760.
"Exploration of S-Matrix Theory".
discusses "hierarchical structure of S-matrix singularities — generate sequence of singularities by "feeding up" from what we already have.
To proceed we shall have to assume properties of crossing, continuation TCP theorem, stable particles etc. Olive's answer is as follows
"We cannot deduce the singularity structure without the fundamental theorems and we cannot deduce the fundamental theorems without

the singular structure, the unitary circle is a fundamental obstacle to the deductive approach to S-matrix theory. ... we -- set up a scheme of successive iterations in terms of the singular structure of the S-matrix. The singular structure that can be deduced is used to derive the fundamental theorems within this "approximation" and then these theorems are used within this "approximation" to generate further singular structure and so on.

Gale, G. "Chew's Monadology."
J. Hist. of Ideas 35 (1974) p. 339 – 348

Gale draws analogies between Chew & Leibniz. Each monad mirrors the whole universe — cp each particle mirrors every other particle.

Everything existed is only possible one by self-consistency. cp Leibniz's many best possible worlds. L required a "Principle of perfection" to provide an overarching criterion of choice between the systems".

Gale stresses Chew says there is only one self-consistent system (or universe) while Leibniz admits the possibility of other self-consistent possible worlds.

Also we criticize Gale because there is no causal interaction between monads.

Chew G.F. Science (1968) 161 p 762–765 "Bootstrap: A scientific Idea?"
Physics Today (1970) 23 Oct issue 23–28.
"Hadron Bootstrap: Triumph or Frustration?"

2 popular articles:

stresses self-consistency
"Nature is as it is because this is the only possible nature consistent with itself" (cf Leibniz all truths are analytic → Sufficient reason.

Bootstrap idea much older than particle physics

Prerequisites for Science
1) 3 D space & time (unaccelerated)
2) separate macroscopic "objects"!
3) gentle forces (e.m) allow one "object" to serve as "measurement"!
4.) Existence of objects yet conflict → Consciousness of measurement

Explanation of all concepts would be unscientific
— some unquestioned framework required.
Partial Bootstrap in line with progress in Science

Existence of consciousness is necessary to self-consistency in whole bootstrap:
opposed views 1) elementary constituents of matter
2) fundamental eq. of motion

P. 80

Bootstrap → no arbitrary parameters

3 rules for hadrons: 1) constituent of a composite
2) exchange → force holding composite together
3) whole composite itself.

But BE early 2 vast mass of correlations

sufficient constraint model rules 1), 2) & 3)

leads fundamental

Tests of Bootstrap 1.) Proof of consequences very different mathematical models

2) hadrons not Regge poles - ~~that~~ to all particles lie on Regge recurrences as prior to neutrons found (1968)

Quarks might also lie on the bootstrap (Re Regge poles

(Chew refers to Chandler & Stapp for relating upper boundary of the Landau branch points to causality (macro))

As part of his course in the philosophy of science Dr Redhead will give a series of lectures on the Philosophy of Space and Time

beginning on Wednesday 5th February at 2.15 pm in the Seminar Room. The course will include ~~an~~ an introduction to the conceptual problems of the special theory of relativity

But even if bootstrap true could then not be a beneath nuclear matter, a basic field field obeying a simple "master eq" of motion (cf. Heisenberg) → not inconsistent with the bootstrap : hadron made of hadrons made of quark field.

difficulty of bootstrap is lack of a definite starting point — no single hadron can be understood without all the others but fields we can approach the complete bootstrap in a series of approximations. covering wider & wider regions of the S-matrix.

Complete bootstrap beyond current physics. includes existence of macroscopic objects from which measurement apparatus is constructed — complete bootstrap involves incorporating the classical concept of observation & possibly even of consciousness.

In Physics Today article,
clear offers the fundamentalist view (F)
and the bootstrapper view (B)

(F) understands nature in terms of fundamentals
(B) . . . . . self-consistency

no arbitrary parameters [illegible] in F [illegible] in B.

[illegible] "fundamental" concept is scientifically acceptable, also to the Bootstrap philosophy

Failure of achieving a partial bootstrap may be because only consistent S-matrix is the full hadron bootstrap with all particles included.

"I would find it a crushing disappointment if in 1980 all of hadron physics could be explained in terms of a few arbitrary entities – we should then be in essentially the same position as in 1930, ... To have learned so little in half a century would to me be the ultimate frustration."